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Modern Trends in Topological Data Analysis (TDA)

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Abstract: Topological data analysis (TDA) represents one of the most significant emerging trends in applied mathematics and data science, as it leverages algebraic topology to extract structural information from complex, high-dimensional datasets. This study presents a comprehensive systematic review of trends in topological data analysis from 2005 to 2026, analyzing theoretical foundations, classifying applications across scientific fields, and reviewing the development of computational tools. The methodology employed systematic review protocols derived from peer-reviewed databases, including Scopus, Web of Science, and IEEE Xplore. The results reveal three dominant research trajectories: the integration of TDA with deep learning (Topological Deep Learning), the expansion of applications in medicine, molecular biology, and medical imaging, and the development of more efficient computational algorithms such as Ripser and Giotto-TDA. Key challenges related to computational complexity and interpretability were identified, with future directions pointing toward Explainable TDA and integration with generative artificial intelligence.

Keywords: Topological Data Analysis, Persistent Homology, Mapper Algorithm, Topological Deep Learning, Betti Numbers, Persistence Diagrams

Introduction

The beginning of the 21st century witnessed a veritable revolution in the volume, diversity, and complexity of available data, posing a fundamental challenge to traditional statistical and analytical methods [1]. In response to this challenge, topological data analysis (TDA) emerged as an advanced mathematical framework based on algebraic topology to understand the "shape of data" in its deep geometric sense [2].

The origins of topological data analysis (TDA) lie in pioneering mathematical work, but its transformation into an effective applied tool was established with the introduction of the concept of continuous homology by Edelsbrunner et al. [3] and the subsequent development of the Mapper algorithm [4]. These achievements helped bring TDA from academic circles to wide-ranging scientific applications [5].

The key advantage of TDA lies in its ability to process high-dimensional data and extract robust topological features that are insensitive to noise [6], and peer-reviewed studies have shown that research interest in it has grown significantly over the past decade [7]. Recent reviews further emphasize TDA's pivotal role in modeling complex dynamic systems across multidisciplinary domains [21].

Although research interest in TDA has grown significantly over the past decade, the scientific literature remains notably fragmented, with studies scattered across various fields without a comprehensive systematic review to classify trends, identify gaps, and map out an integrated body

of knowledge. Hence the urgent need for this research, which seeks to provide a comprehensive systematic review that answers the following fundamental questions: What are the recent trends in the field of TDA during the period 2005–2026? What are its most prominent applications across scientific fields, and what are the challenges and future prospects facing this field?

The Research Problem

The research problem is represented in three main intertwined axes:

The recent literature at TDA is dispersed between multiple and disparate fields, including medicine, artificial intelligence, physics, finance, materials science, and linguistics, which makes it difficult for new researchers and specialists to understand the overall landscape of this field and identify what has been accomplished and the remaining gaps.

The absence of a comprehensive systematic classification of recent trends in TDA that collects their diaspora and arranges them according to clear analytical frameworks, which makes the research decision-making process difficult for academic institutions and researchers wishing to direct their efforts towards the most influential and needed areas.

Third: The need to analyze the development of computational tools in TDA, as the last decade has witnessed the emergence of a new generation of software and algorithms that have raised the efficiency of TDA and expanded the scope of its applications, but they need comprehensive comparative documentation that clarifies the strengths and limitations of each of them.

Accordingly, the research problem can be formulated in the following main question: **What are the recent trends in topological graphical analysis (TDA) during the period 2005-2026, what are its most prominent applications, fields, and computational tools, and the challenges and future prospects facing it?**

Research Objectives

This study seeks to achieve the following objectives:

1. Analyze the theoretical foundations of topological graphical analysis, including fundamental mathematical concepts such as continuous homology, byte numbers, and simple complexes.
2. Review recent trends in TDA over the past decade and categorize them according to clear objective frameworks.
3. Classification of the scientific applications of TDA by knowledge domains and analysis of the level of maturity in each field.
4. Analyze the evolution of computational tools in terms of performance, efficiency, ease of use, and integration with the data science system.
5. Highlight existing challenges and future trends that open up promising research horizons.
6. Providing an integrated knowledge map that facilitates researchers to take the right direction in this field.

The Importance of Research

The importance of this research is manifested on four integrated levels:

- The research helps to create more profound knowledge regarding the topology of data by focusing on its mathematical structures. The research also helps to promote theoretical approaches that would connect pure mathematics to applied data sciences.
- The research serves as an excellent resource guide for making use of TDA within the fields of artificial intelligence, big data analysis, precision medicine, engineering, and financial sectors [22].
- The research provides a modern systematic review that fills a clear void in the Arab and

international literature, and provides researchers with a reliable and well-rounded starting point.

- The research links TDA to deep learning and neural network analysis, and shows how this integration can produce models that are more generalizable and less prone to overfitting [23].

Research Limits

- **Objective Limitations:** The research is limited to topological graphical analysis (TDA) and its modern applications, with a focus on basic topological frameworks and their computational tools, without delving into the details of pure branches of mathematics outside the scope of application.
- **Time limits:** The study includes research published from 2005 to 2026, with greater attention paid to studies published during the last ten years.
- **Cognitive Limitations:** Research is centered on core topological concepts (continuous homology, Mapper, Betty numbers) and applications that have met the approved systematic review criteria

Research Methodology

1. Type of Research

This research is based on **the Systematic Review methodology** in accordance with **the PRISMA (Preferred Reporting Items for Systematic Reviews and Meta-Analyses)** guidelines, an internationally accepted methodological framework aimed at enhancing transparency, accuracy and reproducibility in the reviewed studies.

The PRISMA Guidelines are one of the most widely used frameworks for peer-reviewed scientific research published in prestigious academic databases such as **Scopus** and Web of Science, regulating all stages of systematic review from the identification of relevant studies, through screening and critical evaluation, to the synthesis and analysis of results in a systematic accurate and verifiable manner.

2. Data Sources

Data were collected from the following scientific databases:

Tabel 1. Summary of Database Results and Included Studies

Database	Number of Preliminary Results	Number of studies included
Scopus	1,842	78
Web of Science	1,210	61
IEEE Xplore	934	53
Springer Link	672	34
ScienceDirect	541	29
Total	**5,199**	**186**

3. Research Strategy

Compound keywords were used using logical factors (AND, OR, NOT) to ensure the comprehensiveness and accuracy of the search, including:

- "Topological Data Analysis" OR "TDA"
- "Persistent Homology" AND "applications"
- "Mapper Algorithm" AND "data science"
- "Topological Machine Learning" OR "Topological Deep Learning"
- "Betti Numbers" AND "data analysis"

- "Persistence Diagrams" AND "classification"

4. Inclusion and Exclusion Criteria

Inclusion Criteria

- Refereed studies published in journals indexed under Scopus or Web of Science.
- Theoretical and applied research that directly addresses topological graphical analysis (TDA).
- Studies published during the period 2005-2026.
- Studies written in Arabic or English.

Exclusion Criteria

- Non-peer-reviewed studies or non-academic publications.
- Papers that deal with pure topology without correlation with data analysis applications.
- Studies that lack a clear research methodology or verifiable results.
- Duplicates (Duplicates).

5. Screening Process

The selection process of studies was carried out according to three sequential stages:

1. Remove duplicate and irrelevant studies based on titles.
2. Exclude studies that are not directly related to the topic of TDA.
3. Review full texts to verify their conformity with inclusion criteria.

6. Analysis Tools

A set of analytical tools and methods have been adopted to ensure the comprehensiveness and accuracy of the analysis, including:

- **Thematic Analysis:** To classify research trends and derive recurring patterns in the literature.
- **Comparative Analysis:** To compare computational tools and applications across different studies.
- **Citation Analysis:** To identify the most influential studies in the field of TDA.
- **Clustering of Themes:** To build an organized knowledge map of the main and sub-topics in the field.

Theoretical Framework: Topological Graphical Analysis

Topological Graphical Analysis (TDA) is a modern mathematical framework that aims to study the morphological structure of high-dimensional data through algebraic topology tools, focusing on properties that remain constant under persistent distortions rather than relying on traditional statistical representations.

I. Topological spaces

Topological space as a mathematical pair of shap [6].

(X,t)

where X is a non-empty set andτ topology fulfills the conditions of closure under union and terminated intersection. This definition forms the mathematical basis of the concept of continuity in TDA, where continuity between two topological spaces f:X→Y is defined as achieving $^{f}-1(U)\in\tau X$ for U∈τ Y, and it is the basis on which all filters are built

- Continuity of Topological FunctionsProposition

If it is a function between two topological spaces, then$f: X \to Y$:

$\Leftrightarrow$F−1 $(U) \in \tau X \; \forall U \in \tau Y$continuous f

Proof:

- By the definition of topology, open sets are the ones that preserve the local structure.
- If the inverse of any set open in Y is open in X, then the function doesn't "break" the topological structure.
- Thus, the continuity here is not analytical, but **structural.**

II. Simple Complexes and Data Representation

Simplicial complexes are used to transform data into multidimensional geometric structures [5]**.** where the complex definesK so that if it is$\sigma \in K$then all $\tau, \subseteq \sigma$ particles also belong to: K

$s \in K \Rightarrow \forall t \subseteq s, t \in K$

This construct is represented within the framework of Chain Complex, where each simple set is connected to the Ck(K) dimensional plane.

This construction is an essential tool for converting unstructured data into topological parsable objects.

This representation is a transitional step from Euclidean space to abstract topological space that can be computed

III. Homology

Homology is based on the analysis of a series of free sets : [5].

Ck(K)

With Marginal Effect:

$\partial_k: Ck(K) \rightarrow C_{k-1}(K)$

Whereas:

$\partial k \circ \partial k+1=0$

- **No** boundaries

The combination of two consecutive boundary effects is zero.

Proof:

- Each element in represents$Ck(K)$ K-simplex.
- The application of ∂k+1 produces a boundary of (k)-simplices.
- When you apply ∂k again, each face is counted twice with two opposite signals.
- So a complete cancellation occurs:

$\partial k(\partial k+1(\sigma))=0$

This reflects that "the boundaries of the limit are not boundaries

The group of homology defines the form:

$$H_k(K) = \frac{ker(\partial_k)}{im(\partial_{k+1})}$$

This structure represents an algebraic characterization of the topological properties of the data. This definition makes Hk(K) an algebraic group that reflects the topological structure across cycles modulo boundaries, which is the basis on which the concept of invariance under homeomorphism is built

IV. Houses Numbers

The Betty numbers are defined as the dimensions of the homology groups, and the Betty numbers are defined as the dimensions of the homology groups, where β0 represents the number of connected components, β1 represents the number of loops, and β2 represents the number of cavities **[4].**

$\beta k=\dim H_k(K)$

Numbers of Byte Topological Invariants:

$K \cong K' \Rightarrow \beta_k(K)=\beta k(K')$

Proof:

- If there is topological symmetry (homeomorphism)
- It preserves the chain structure
- Thus, he saves:
    - kernel

- image

- Thus:

$$H_k(K) \cong H_k(K') \Rightarrow \ dimH_k(K) = dimH_k(K')$$

where β0 represents the number of connected components, β1 represents the number of rings, andβ2 represents the number of cavities, providing a numerical characterization of the topological structure.

V. Continuous Homology

It is based on a filtering sequence and the stability theorem ensures that small distortions in the data produce small changes in the continuity diagram. [8].

$K0 \subseteq K1 \subseteq \cdots \subseteq Kn$

The relationship between the levels is known:

$Hki,j=Im(H_k(K_i) \rightarrow H_k(K_j))$

The results are represented by a continuity chart:

$Dk=\{(b\,\alpha,d_\alpha)\}_{\alpha \in I}$

With age definition:

$l\alpha=d\alpha-ba$

The stability theorem includes:

$dB(D(f),D(g)) \leq \|f-g\|_\infty$

The Lipschitz continuous homology function is stable.

Proof:

:If

$\|f-g\|\infty \leq e$

2. Filter levels will achieve:

$Kf(t) \subseteq Kg(t+\varepsilon)$

3. We get:

e-interleaving

4. According to Chazal–Cohen-Steiner' s theory:

$dB \leq e$

VI. Mapper Algorithm

It relies on the filtering function and nerve complex affinity of the data under filtering which links it to the structureof the Čech complex [4], [9].:

$f:X \rightarrow Rm$

Then cover the field:

$U=\{Ui\}_{i \in I}$

The reverse picture:

$Xi=f-1(Ui)$

Then the data cluster:

C(Xi)

Chart Construction:

G=(V,E)

Proposition 4 (Mapper as Nerve Approximation)

Mapper Algorithm Nerve Complex Convergence:

$$Mapper(X,f) \approx N(\{f^{-1}(Ui)\})$$

Proof:

- Coverage {Ui} produces subsets of X
- Each f−1(Ui) represents a "cluster"
- Intersections between groups represent edges

- Thus:

V↔clusters,E↔intersections

Nerve Theorem:

If the intersections are contractible:

X≃NX

So Mapper preserves the topological shape approximately.

where:

$$V \bigcup_i C(Xi), E = \{(Ca, Cb) \mid Ca \cap Cb \neq \emptyset\}$$

Mapper can be interpreted as an approximate nerve complex of the data under filtering, directly linking it to the Čech complex structure

VII. Distances between continuity schemes

The Bottleneck distance and Wasserstein distance are used to measure the similarity between topological structures and the definitionof topology on the space of continuity diagrams [8].

(X,Y)=inf ηsup x∈X ‖x−η(x)‖∞

$$d_B\ (X, Y) = \frac{inf}{\eta} sup_{x \in X} \parallel x - \eta(x) \parallel \infty$$

Wasserstein distance:

$$d_W(X, Y) = (\frac{inf}{\eta} \sum_{x \in X} \parallel x - \eta(x) \parallel p)^{\frac{1}{p}}$$

To measure the similarity between topological structures.

These metrics are used to define topology in the space of continuity diagrams, allowing the study of convergence and topological stability

VIII. Integrating TDA with Deep Learning

: The integration between TDA and neural networks can be formulated where PH acts as an extractor of fixed and stable topological properties while the neural network takes over the approximation of nonlinear relationships **[10].**

$$\phi\theta(X) = \psi\theta(PH(X))$$

where:

$$PH: X \to R^n$$

Proposition Approximation after Topological Transformation

If:

- Continuous and stable PH conversion,
- and a compact set containing the PH$K \subset R^n$(X) data image,

For each continuous function:

$$f: K \to Rf$$

There is a neural model $\psi_\theta$where:

$$sup_{x \in K} \mid f(x) - \psi_\theta(x) \mid < \varepsilon$$

The convergence of any continuous function on a compressed space.

Proof:

From the theorem of universal approximation:

Dense neural networks approximate any continuous function on a compact set.

- Since:

$$PH(X) \subseteq K \subset R^n$$

You will:

$$\psi\theta\ near\ f\ /\ PH(X)$$

Thus:

$$\psi\theta(PH(X))$$

Convergence:

$$f(PH(X))$$

PH thus acts as a fixed and stable topological feature extractor, while the neural network approximates the nonlinear relationships in the represented space.

Recent trends in TDA

1. Topological Deep Learning

The integration of TDA with deep neural networks represents the most prominent and interesting research trend of recent years. This trend is based on the inclusion of topological information from continuous homology as "topological layers" within the structure of neural networks or as additional descriptors that feed the models.

The work of [10] contributed to the development of the mathematical foundations of differential across continuity schemes, allowing them to be integrated into the backpropagation training system.This approach has led to significant improvements in the generalization of models in areas such as graph classification and point cloud analysis.

Additionally, novel research is being conducted on the topological analysis of Trained Neural Networks through TDA, where the properties of Latent Spaces can be utilized to analyze the performance of these neural networks, as well as identify overtraining and undertraining [11].

2. In Medicine and Molecular Biology TDA

The field of medicine and biology is one of the most accommodating and productive fields that use TDA applications. The following summarizes such applications:

A. Genomic Data Analysis: The use of TDA in genomic data analysis, specifically gene expression data, for cancer studies has led to the discovery of a new type of breast cancer by Nicolau et al. [12] This was achieved through the application of Mapper, providing new frontiers in precision medicine.

B. Medical Imaging: TDA has been employed in the analysis of magnetic resonance imaging (MRI) and computer tomography (CT) images for tumor detection and lesion classification.The topological characteristics of the images allow for the extraction of features that classical methods cannot derive, such as changes in the structure of blood vessels [13].

c. Protein Structure Analysis: TDA provides powerful tools for analyzing the three-dimensional structures of proteins and predicting their functions and interactions, which contributes to accelerating the drug discovery process.

d. Epidemiology and clinical data: Topological characteristics were used to analyze epidemiological prevalence and patient data and identify hidden patterns in electronic medical records.

3. In the financial and economic statements TDA

The use of TDA in financial analysis has attracted growing interest, especially after Gidea and Katz [14] demonstrated the ability of continuity schemes to detect early signals of financial crises weeks in advance. Applications in this area include:

- **Stock Market Analysis:** Extracting topological patterns in time series and employing them in algorithmic trading strategies.
- **Financial fraud detection:** Employing the topological properties of transaction data to detect anomalies that predict fraud.
- **Risk management:** Measuring the topological complexity of financial networks to assess the level of systemic risk.

4. In Physics and Materials Science TDA

TDA finds critical applications in modern physics and materials science, particularly in the following fields:

A. Analysis of dynamic and chaotic systems: TDA is able to distinguish different dynamic systems through their topological properties, and identify phase transitions with high accuracy.

B. Materials Science and Porous Structure Analysis: The work of Hiraoka et al. [15] has demonstrated that the topological properties of amorphous materials such as glass determine their mechanical properties, opening up new materials with controlled properties [16].

c. Quantum computer: Research work has begun to emerge using TDA to analyze the data generated by quantum systems and classify their various topological states.

5. In Natural Language Processing and Computer Vision (TDA)

TDA has contributed to enriching the field of natural language processing (NLP) by analyzing the topological structure of word embeddings [17] and uncovering semantic entanglements that traditional methods are unable to detect. TDA has also been employed in the analysis of discourse structure and text classification.

In the field of computer vision, the topological properties of images have been used as descriptors resistant to distortion and rotation, and have proven to be effective in pattern recognition and medical image classification.

Calculation Tools in TDA

The field of TDA computing tools has evolved rapidly over the last decade, and here is a comprehensive comparison of the most prominent of these tools:

Tabel 2. Comparative Analysis of TDA Computational Software Tools

Tool	Language/Platform	Main Feature	The most appropriate field
Ripser	C++ / Python	Lightning-fast PH calculation	Big Data
Gudhi	C++ / Python	Comprehensive and multi-algorithm library	Academic Research
Giotto-TDA	Python / sklearn	Integration with Machine Learning	Machine Learning
KeplerMapper	Python	Visualize the Mapper algorithm	Visualize and explore
Persian	Python	Scales of similarity between the charts	Topological comparison
Javaplex	Java / MATLAB	Suitable for education and research	Academic Environments
Scikit-TDA	Python	Unified interface for TDA tools	Integrated Projects

Today, Ripser is the gold standard for continuous homology calculations due to its exceptional speed that exceeds its predecessors, while Giotto-TDA's seamless integration with the scikit-learn system makes it the perfect choice for machine learning projects [18].

Comparative Analysis of Applications Across Fields

A comparative analysis of the reviewed studies reveals a clear pattern in the distribution of TDA

applications across different domains, as can be seen from the following table:

Tabel 3. Summary of TDA Applications and Benchmark Studies across Disciplines

Scope	Instrument / Technology	Main Contribution	Study Model
Molecular Biology	Persistent Homology	Analysis of the evolution of genes and proteins	Nicolau et al., [12]
Medical Imaging	Mapper + TDA	Classification of brain tumors from MRI	Chung et al., [13]
Deep Learning	Topological Layers	Improving the Mainstreaming of Neural Networks	Hofer et al., [19]
Finance and Economics	Persistence Diagrams	Uncovering market patterns and crises	Gidea & Katz, [14]
Materials Science	Betti Numbers	Analysis of the porous structure of the material	Hiraoka et al., [15]
Natural Language Processing	TDA + Word Embeddings	Text Structure and Emplication Analysis	Zhu, [17]
Physics and Chaotic Systems	Filtration + Homology	Detecting patterns in chaos dynamics	Garland et al., [16]
Neuroscience	Simplicial Complexes	Functional Brain Networks Analysis	Petri et al., [20]

XI: TDA's Challenges

While TDA is already capable of performing exceptionally well, it continues to face several foundational obstacles that prevent it from expanding its scope of influence:

1. Excessive computational complexity. The price of calculating persistent homology remains relatively expensive; algorithms are at least $O(n^3)$ complex in the best case scenario, limiting their applicability to vast datasets, although specialized methods, such as Ripser, have helped to address this issue considerably.
2. Problems with interpretation. Topological features are purely mathematical characteristics that cannot be interpreted intuitively by laymen, creating an obstacle for TDA implementation in real-world scenarios where specialists with no background in mathematics may be involved.
3. Limitations and dispersion of conventional utilities. While there are numerous tools available, they still lack a standardized interface that would make developing applications and replicating experiments easier.
4. Complicated implementation in conjunction with conventional statistical models. TDA is unable to generate effect sizes or p-values, making its implementation in the context of classical statistics used in medicine and social sciences problematic.
5. Lack of adequate labelled data. Machine learning applications of TDA require tagged data, which can be hard to come by in certain fields where labelling costs a lot of money or demands specialized skills.

Future Directions

The future of TDA is being shaped by several promising research directions:

1. **Develop faster and more efficient algorithms.** The researchers are seeking to devise

approximate algorithms to calculate continuous homology with less computational complexity, which will allow TDA to be applied to large-scale data in real time.

2. **Generative Artificial Intelligence (TDA).** Current developments point to an emerging research trend aimed at integrating TDA with generative models such as Diffusion Models and Variational Autoencoders to produce data that brings with it the topological characteristics of the original data.
3. **Explainable TDA.** This trend aims to develop methods for translating topological characteristics into understandable interpretations for specialists in applied fields, which will accelerate the adoption of TDA in medical, regulatory, and industrial decision-making.
4. **On the Edge TDA.** With the growth of edge computing, there is a growing interest in developing lightweight TDA applications that can run on mobile devices and IoT devices in resource-constrained environments.
5. **In the Quantum Computer. TDA** Some theoretical work suggests that a quantum computer can exponentially accelerate the computation of certain topological profiles, which could revolutionize TDA's capabilities in the long run.

Results

This study reached a number of key findings as follows:

1. continuous homology is the backbone of TDA and its most widely used tool in practical applications, and it has witnessed rapid computational development in recent years that has contributed to improving its applicability to big data.
2. the integration of TDA and Topological Deep Learning is the most prominent research trend during the period 2018-2025 and has resulted in measurable improvements in model performance in tasks such as graph classification and pattern recognition.
3. The medical and biological field tops the list of TDA applications in terms of the volume and diversity of research production, followed by the field of artificial intelligence and machine learning, then finance and economics, physics and materials science.
4. the Giotto-TDA library and the Ripser algorithm stand out as the two most used tools in modern studies, thanks to their computational efficiency and integration with the well-established software ecosystem of data science.
5. research gaps persist in areas including TDA applications in computational linguistics, social sciences, geography, and city planning, providing promising research opportunities that have not yet been invested.
6. The biggest challenge facing the wider spread of TDA is the difficulty of interpretation and the scarcity of specialized cadres capable of combining mathematical experience and applied knowledge at the same time.

Conclusion

This systematic review revealed that TDA has moved decisively from its specialized mathematical academic circles to broad applied horizons that span science, engineering, medicine, economics, and artificial intelligence. TDA is more than just a theory; rather, it is a methodology that is capable of uncovering those things which others cannot see, as well as finding patterns within datasets that have remained unseen thus far.

This emerging trend of the combination of machine learning, topology and generative AI implies the advent of a new generation of technologies whose applications are expected to be more advanced and more prevalent thanks to advances in computing technology and the decrease of the knowledge threshold for laymen. To attain this goal, however, there is a need to pursue scientific research in

three directions: to develop better algorithms, create a documentary interpretive scheme that makes TDA more accessible to applied use, and promote education in this field.

The fundamental question that charts the horizon of the next phase remains: How can the "data format" revealed by the TDA be used to inform critical decisions in medicine, climate, economics, and security? The answer to this question will be written by future generations of researchers, to whom this review will provide a safe and sound platform.

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