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Fluid Motion Behaviour Under External Magnetic Field

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Abstract: This paper investigates the effects of an external magnetic field on fluid motion behaviour. One dimensional Navier-Stokes equation along with added magnetic field equation has been employed for this purpose. The fluid assumed here is a hypothetical ferromagnetic fluid having magnetic fluid dynamics. A steady, laminar, viscous fluid has been considered with magnetic field applied in a direction normal to the flow of the fluid. The numerical outcomes and their dependence on various parameters of fluids have been discussed thoroughly.

1. Introduction

The dynamics of fluid motion under the influence of magnetic fields provides the basis for magnetohydrodynamics (MHD),the field that is constituted of fluid dynamics and electromagnetism. In current-conducting fluids, for example plasma, magnetic liquids, molten metallic fluids and ionic solutions, Interplay between moving fluids and magnetic forces generates rich and challenging behavior patterns. [1]. When a conducting fluid moves through an external magnetic field, electric currents are induced according to electromagnetic induction. The flow of electric charge interacting with the magnetic field, create a force throughout the fluid known as Lorentz force, which can considerably alter the velocity, stability, and transport properties of the fluid. This coupling between the magnetic and velocity fields leads to a rich variety of phenomena, including flow suppression and even reversal, turbulence, boundary layer modification, and magnetically driven instabilities etc [2]. Traditionally MHD has been used in complex compound fluids, plasma physics and astrophysical systems. Yet, its relevance at smaller scales is growing. Despite its broad applicability, the mathematical description of MHD flows remains highly challenging due to the nonlinear coupling between the Navier-Stokes equations and Maxwell's equations. Analytical solutions are limited to simplified configurations in this paper where we have limited ourselves to the 1-Dimensional version of the Navier Stokes equations, while realistic problems often require numerical approaches and advanced computational techniques which will be looked into in the immediate future .

In this research effort, we investigate the behavior of moving fluids subjected to an external magnetic field within a non-relativistic MHD setting. Emphasis is placed on understanding how magnetic field strength, boundary conditions, and fluid properties influence flow characteristics. Next we move onto the mathematical formalism, numerical methods and results and discussion sections. These would be followed by the conclusion and the bibliography [3, 4].

2. Mathematical Formalism

We analyze a single-direction flow of a conducting, viscous fluid subject to an applied magnetic field, using a magnetohydrodynamic (MHD) approach. The motion is taken to be along the x-axis, with all physical quantities depending on space x and time t only [5, 6, 7].

2.1. Governing Equations

The evolution of the system is controlled by coupled Navier–Stokes and Maxwell's equations [8, 9, 10].

2.1.1. Continuity Equation

The governing expression for mass conservation is:

$$\partial\rho\backslash\partial t + \partial(\rho u)/\partial x = 0 \quad (1)$$

in which ρ(x, t) represents the density of the fluid and u(x, t) denotes its velocity distribution.

2.1.2. Momentum Equation

The momentum conservation equation including the Lorentz force is:

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{\partial x^2} + \left(\frac{1}{\mu_0}\right)B\frac{\partial B}{\partial x} \quad (2)$$

In this expression, p(x, t) represents the fluid pressure, μ denotes the coefficient of dynamic viscosity, B(x, t) corresponds to the magnetic field intensity, and μ_0 stands for the magnetic permeability of the vacuum.

2.1.3. Induction Equation

The time development of the magnetic field is governed by the induction relation:

$$\frac{\partial B}{\partial t} + \frac{\partial}{\partial x}(uB) = \eta\frac{\partial^2 B}{\partial x^2} \quad (3)$$

in which $\eta = 1/(\sigma\mu_0)$ denotes the magnetic diffusivity, with σ representing the electrical conductivity

2.1.4. Equation of State

The model becomes complete with the addition of an equation of state. For a compressible fluid, one may use:

p = p (ρ,T) (4)

or, assuming a perfect gas model:

p = ρ RT (5)

where T is temperature and R is gas constant

2.2. Incompressible Limit

in the case of an incompressible fluid, the density remains unchanged:

ρ = constant (6)

and the mass conservation relation simplifies to:

$\partial u/\partial x = 0$ (7)

2.3. Simplified 1D MHD System

Under the assumptions of constant density and negligible pressure gradient, the model equations simplify into:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2} + \left(\frac{1}{\rho\mu_0}\right)B\frac{\partial B}{\partial x} \quad (8)$$

$$\frac{\partial B}{\partial t} + u\frac{\partial B}{\partial x} = \eta\frac{\partial^2 B}{\partial x^2} \quad (9)$$

where ν = μ/ρ is the kinematic viscosity.

2.4. Initial and Boundary Conditions

suitable starting and boundary constraints are required to complete the formulation.

conditions prescribed at the initial time:

u (x,0) = $u_0(x)$ (10)

B (x,0) = $B_0(x)$ (11)

Boundary conditions (example for finite domain 0 ≤ x ≤ L):

U (0,t) = u_L , u(L,t) = u_R, (12)

B (0,t) = B_L, B(L,t) = B_R. (13)

2.5. Dimensionless Form

Introducing characteristic scales: length L, velocity U, magnetic field B_0, and time L/U, the variables are expressed in nondimensional terms:

$$x^*=\frac{x}{L}, \quad t^*=\frac{tU}{L}, \quad u^*=\frac{u}{U}, \quad B^*=\frac{B}{B_0} \quad (14)$$

The dimensionless equations become:

$$\frac{\partial u^*}{\partial t^*}+u^*\frac{\partial u^*}{\partial x^*}=\left(\frac{1}{R_e}\right)\frac{\partial^2 u^*}{\partial x^{*2}}+\left(\frac{1}{M^2}\right)B^*\frac{\partial B^*}{\partial x^*}$$

$$\frac{\partial B^*}{\partial t^*}+u^*\frac{\partial B^*}{\partial x^*}=\left(\frac{1}{R_m}\right)\frac{\partial^2 B^*}{\partial x^{*2}} \quad (15)$$

where the dimensionless numbers are:

Re = UL/ν (Reynolds number)

Rm = UL/η (Magnetic Reynolds number)

M = U/V_a (Alfven Mach number) (16)

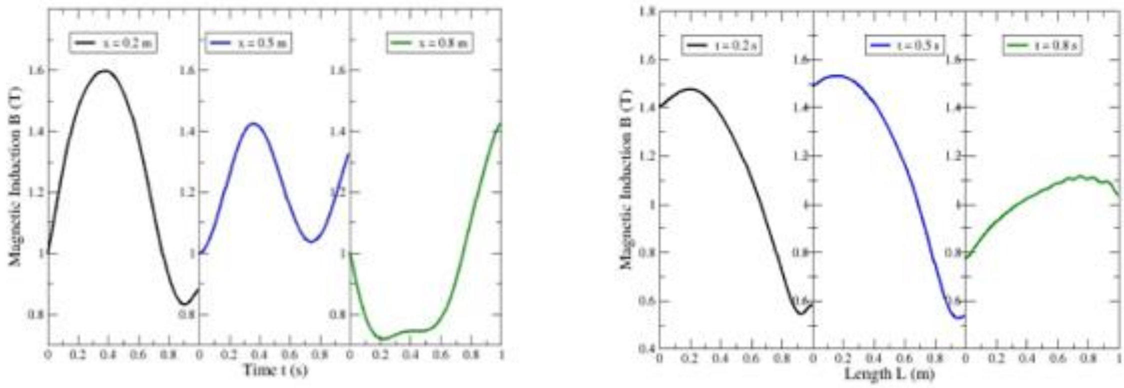
and $V_A = {B_0}/{\sqrt{\rho\mu_o}}$ = is the Alfven velocity.

2.6. Remarks

The formulation reveals a nonlinear interaction linking the velocity and magnetic fields. the balance between viscous forces and magnetic influences is characterized by the Reynolds number and the magnetic Reynolds number, respectively.The Alfven Mach number characterizes the strength of magnetic forces relative to inertial effects [11, 12, 13].

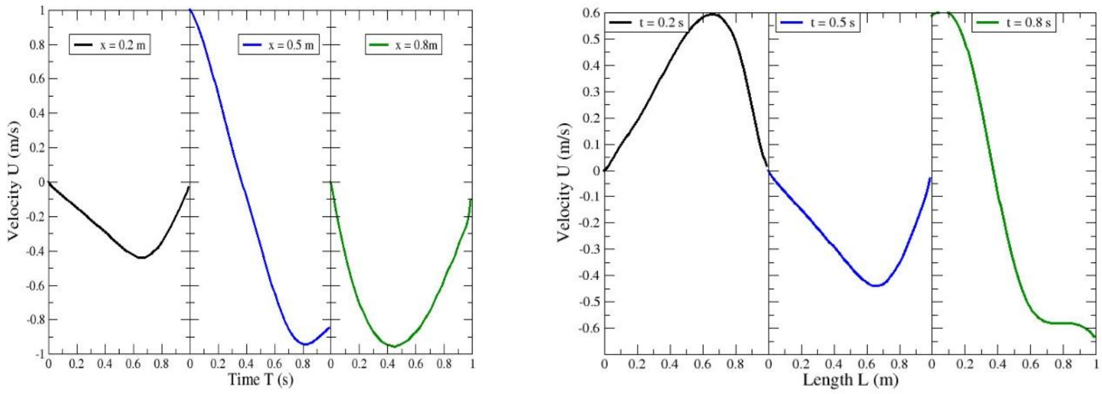
3. Results and Discussions

The fig. 1 shows the magnetic field induction profile with respect to time at fixed length positions and with respect to position at fixed time-stamp. Although the magnetic field is kept initially at 1 T, the field varies with length and time considerably. Therefore the notion of fixed magnetic field for entire flow channel may not show the full picture [12, 14, 15, 16].



[Figure 1: Magnetic field induction B profile as function of (left) Time (right) Length. Here both time and length have been scaled]

The fig. 2 shows the velocity profile with respect to time at fixed length positions and with respect to position at fixed time-stamp. The fluid velocity although starts with a positive value but as per the plot, it changes direction (shown by the negative values) within an interval in both length and time configuration. This we feel is due to the changing magnetic field along space and time. It may also indicate a rise in non-laminar flow such as turbulences etc. This interesting feature may be highlighted in the higher dimension flow pattern which we will report in our next report soon [17, 18].



[Figure 2: Fluid velocity u profile as function of (left) Time (right) Length. Here both time and length have been scaled]

4. Conclusions

From these results we find that the magnetic field does not remain steady. It oscillates. At different spatial points, the oscillations are out of phase. Near the origin, the field rises quickly, reaches a peak, and then declines. Farther away, the response is delayed and weaker. This indicates wave-like propagation combined with diffusion. Neither effect alone is sufficient to explain the observed behavior. At early times, the magnetic field varies smoothly across space. Gradients are gentle. But as time progresses, distortions appear, peaks shift and profiles become steeper. The field begins to reorganize itself. This redistribution is a clear sign of nonlinear coupling between field and flow. Velocity shows a strong dependence on time. Initially, motion is mostly forward. Gradually, negative regions appear. In some locations, reverse flow becomes dominant.

The study demonstrates that magnetic fields significantly alter fluid dynamics, even in a simplified non-relativistic system. The presence of a time-dependent magnetization term proves essential. Without it, key features would be lost.

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