



A Review on the Integral Transforms SEE, g, ZZ, ZMA, AMJ, GFI

Maha S ALibrahimi

Al-Furat Al-Awsat University – Najaf Technical Institute, Najaf, Iraq

Abstract: This article describes a new type of integral transformations and their fundamental attributes. These transformations have been beneficial to scientific research in the resolution of differential equations, additional equations, and systems of differential equations. The Laplace transform was one of the earliest transforms that were employed to address mathematical issues, from this point forward, several other transforms have been developed, primarily based on the Laplace transform.

Keywords: SEE I Transform, g- transform, ZZ transform, ZMA Transform, AMJ, GFI.

1-Introduction

Many scientists utilize transformations that are integral to address difficult issues in natural and social sciences. Integral transformations are used to address differential and integral equations, simplify difficult problems, and explore complex systems. They help convert these systems from the temporal domain to the field of signal processing, particularly in the audio and communication fields[10, 11]

Other important transforms include the Shehu transform, Sumudu transform, Elzaki transform, Mellin transform, Temimi transform, and Fourier transform. [19,5,2,6 ,7 ,8]

Pierre-Simon Laplace (1782) popularized the concept of transformation, which is one of the most celebrated mathematical, physical, and engineering transformations. [3,4]

Other integral transformations are employed to address the heat equation, the gamma equation, and the Bessel equation. [20,12 ,13]

Eventually, several fundamental transformations have been conceived of, the majority of which are intended for the people who conceived of them. This article will explore some of the aforementioned transformations and discuss their properties in chronological order.

2. SEE Integral Transformation[15]

The integral transform by Sadiq, Emad A. Kuffi, and Eman is called the SEI integral transform, and its definition is as follows:

$$\mathfrak{s}\{d(t)\} = \frac{1}{\hbar\omega} \int_0^{\infty} d(t) e^{\hbar t} dt = T(\hbar), \omega \in \delta, \hbar \in [\gamma_1, \gamma_2]$$

SEE Transform of certain simple functions are:

$$1) \mathfrak{s}\{t^n\} = \frac{n!}{\hbar^{\nu+n+1}}, n \in \mathbb{N}$$

$$2) \mathfrak{s}\{e^{\hbar t}\} = \frac{1}{\hbar^{\nu}(\hbar - \hbar)}, \hbar \in \mathbb{R}$$

$$3) \mathfrak{S}\{\sin \sin \ell t\} = \frac{\ell}{\mathfrak{f}^v (\mathfrak{f}^2 + \ell^2)}$$

$$4) \mathfrak{S}\{\cos \cos \ell t\} = \frac{\mathfrak{f}}{\mathfrak{f}^v (\mathfrak{f}^2 + \ell^2)}$$

$$5) \mathfrak{S}\{\sinh \sinh \ell t\} = \frac{\ell}{\mathfrak{f}^v (\mathfrak{f}^2 - \ell^2)}$$

$$6) \mathfrak{S}\{\cosh \cosh \ell t\} = \frac{\mathfrak{f}}{\mathfrak{f}^v (\mathfrak{f}^2 - \ell^2)}$$

SEE Transform of derivatives:

Let $\mathfrak{S}\{d(t)\} = T(\mathfrak{f})$

$$i) \mathfrak{S}\{d'(t)\} = \mathfrak{f} T(\mathfrak{f}) - \frac{1}{\mathfrak{f}^v} d(0)$$

$$ii) \mathfrak{S}\{d''(t)\} = \mathfrak{f}^v T(\mathfrak{f}) - \frac{1}{\mathfrak{f}^v} d'(0) - \frac{1}{\mathfrak{f}^{v-1}} d(0)$$

$$iii) \mathfrak{S}\{d^{(n)}(t)\} = \mathfrak{f}^v T(\mathfrak{f}) - \sum_{\eta=0}^{n-1} \frac{1}{\mathfrak{f}^{v-\eta}} d^{(v-\eta-1)}(0), n=1,2,3,\dots$$

3-g- transform [14]

Yusra AL-Ameri and Methaq Hamza introduced a mutation that was significant in nature. This was referred to as the g-transform. This alteration is recorded for the function $d(t)$ as follows:

$$\mathfrak{Chk}_\zeta \{d(t)\} = \mathfrak{z}^k \int_0^\infty d(t) e^{-\mu \sigma t} dt = d_h k(\zeta), \zeta > 0$$

The g-translate of some fundamental operations is:

$$1) \mathfrak{Chk}_\zeta \{t^n\} = n! h^{-(n+1)k}, n \in \mathbb{N}$$

$$2) \mathfrak{Chk}_\zeta \{e^{\sigma t}\} = \frac{q^\sigma}{q^\sigma - \sigma}, \sigma \in \mathbb{R}$$

$$3) \mathfrak{Chk}_\zeta \{\sin \sin (\sigma t)\} = \frac{\sigma q^\sigma}{q^{2k+\sigma^2}}, \sigma \in \mathbb{R}$$

$$4) \mathfrak{Chk}_\zeta \{\cos \cos (\sigma t)\} = \frac{q^{\sigma+k}}{q^{2k+\sigma^2}}$$

$$5) \mathfrak{Chk}_\zeta \{\sinh \sinh (\sigma t)\} = \frac{\sigma q^\sigma}{q^{2k-\sigma^2}}$$

$$6) \mathfrak{Chk}_\zeta \{\cosh \cosh (\sigma t)\} = \frac{q^{\sigma+k}}{q^{2k-\sigma^2}}$$

g-Transformation of derivatives:

Let $\mathfrak{Chk}_\zeta \{d(t)\} = d_h k(\zeta)$, then

$$i) \mathfrak{Chk}_\zeta \{d'(t)\} = \mathfrak{z}^k d_h k(\zeta) - \mathfrak{z}^k d(0)$$

$$ii) \mathfrak{Chk}_\zeta \{d''(t)\} = \mathfrak{z}^{2k} d_h k(\zeta) - \mathfrak{z}^k \{\mathfrak{z}^k d(0) - d'(0)\}.$$

$$iii) \mathfrak{Chk}_\zeta \{d^{(n)}(t)\} = \mathfrak{z}^{nk} d_h k(\zeta) - \mathfrak{z}^k \sum_{\xi=0}^{n-1} \mathfrak{z}^{(k-\xi-1)} d^{(k-\xi-1)}(0), n=1,2,3,\dots$$

4-ZZ Integral Transformation[9]

In his effort to create the Zain UII, Abadin Zafar introduced a transform called the ZZ transform. This transformation is documented for the function $\varsigma(t)$ as follows:

$$\mathcal{H} \{ \zeta(t) \} = \int_0^\infty \zeta(\beta t) dt = Z(\beta, \delta), t \geq 0, {}^\circ C_1 \leq \beta \leq {}^\circ C_2$$

The transformations of the ZZ type are fundamental to many functions.:

$$1) \mathcal{H} \{ t^n \} = \frac{n! \beta^n}{\beta^n}, n \in \mathbb{N}$$

$$2) \mathcal{H} \{ e^{\beta t} \} = \frac{\beta}{(\beta - \delta)}, \delta \in \mathbb{R}$$

ZZ Transform of derivatives:

Let $\mathcal{H} \{ \zeta(t) \} = Z(\beta, \delta)$, then

$$i) \mathcal{H} \{ \zeta'(t) \} = \frac{\delta^2}{\beta^2} Z(\beta, \delta) - \frac{\delta \zeta(0)}{\beta^2}$$

$$ii) \mathcal{H} \{ \zeta''(t) \} = -\frac{1}{\beta^2} \zeta'(0) - \frac{1}{\beta^2} \zeta(0)$$

$$iii) \mathcal{H} \{ \zeta^{(n)}(t) \} = \frac{1}{\beta^n} T(\beta) - \sum_{\eta=0}^{n-1} \frac{1}{\beta^{n-\eta}} \zeta^{(\eta)}(0), n=1,2,3,\dots$$

ZMA Transform[16]

Zainab M-Alwan proposed a singular transform called the ZMA transform, which is concerned with the function $g(t)$.

$$\mathcal{A} \{ g(t) \} = \int_0^\infty g(\epsilon t) e^{-\frac{t}{\epsilon}} dt = \text{ZMA}(\epsilon, \beta)$$

The transform of the ZMA function is part of the fundamental functions.:

$$1) \mathcal{A} \{ t^n \} = n! \beta^n \epsilon^n, n \in \mathbb{N}$$

$$2) \mathcal{A} \{ e^{\epsilon t} \} = \frac{1}{1 - \epsilon \beta \epsilon}, \epsilon \in \mathbb{R}$$

$$3) \mathcal{A} \{ \sin \sin \epsilon t \} = \frac{\epsilon \beta \epsilon}{1 + \epsilon^2 \beta^2 \epsilon^2}$$

$$4) \mathcal{A} \{ \cos \cos \epsilon t \} = \frac{1}{1 + \epsilon^2 \beta^2 \epsilon^2}$$

ZMA Transform Derivatives:

Let $\mathcal{A} \{ g(t) \} = \text{ZMA}(\epsilon, \beta)$, then

$$i) \mathcal{A} \{ g'(t) \} = \frac{1}{\beta \epsilon} \text{ZMA}(\epsilon, \beta) - \frac{1}{\beta \epsilon} \psi(0)$$

$$ii) \mathcal{A} \{ g''(t) \} = \left(\frac{1}{\beta \epsilon} \right)^2 \text{ZMA}(\epsilon, \beta) - \frac{1}{\beta \epsilon} \psi(0) - \left(\frac{1}{\beta \epsilon} \right)^2 \psi(0).$$

$$iii) \mathcal{A} \{ g^{(n)}(t) \} = \left(\frac{1}{\beta \epsilon} \right)^2 \text{ZMA}(\epsilon, \beta) - \sum_{\eta=0}^{n-1} \frac{1}{\beta^{n-\eta}} \zeta^{(\eta)}(0), n=1,2,3,\dots$$

5-AMJ Transformation[17]

Adil Mousa popularized a transform that was integral, the AMJ transform. This transformation is documented for the function $d_\tau(\tau)$ as follows:

$$\mathcal{A} \{ d_\tau(t) \} = \int_0^\infty d_\tau \left(\frac{\tau}{\rho} \right) e^{-\rho \tau} d\tau = J(\rho), \rho > 0, 0 < \vartheta < \alpha$$

The AMJ transformation of some basic abilities is:

$$1) \mathcal{A}\{t^n\} = \frac{n!}{s^{2n+1}}, \quad n \in \mathbb{N}$$

$$2) \mathcal{A}\{e^{qt}\} = \frac{s}{s^2 - q}, \quad q \in \mathbb{R}$$

$$3) \mathcal{A}\{\sin \sin qt\} = \frac{qs}{s^4 + q^2}$$

$$4) \mathcal{A}\{\cos \cos qt\} = \frac{s^3}{s^4 + q^2}$$

$$5) \mathcal{A}\{\sinh \sinh(qt)\} = \frac{qs}{s^4 - q^2}$$

$$6) \mathcal{A}\{\cosh \cosh(qt)\} = \frac{s^3}{s^4 + q^2}$$

AMJ Transform of Derivatives:

Let $\mathcal{A}\{d_\tau(t)\} = J(q)$, then

$$i) \mathcal{A}\{d_\tau'(t)\} = s^2 J(q) - q d_\tau'(0)$$

$$ii) \mathcal{A}\{d_\tau''(t)\} = s^4 J(q) - s^3 d_\tau(0) - q d_\tau'(0)$$

$$iii) \mathcal{A}\{d_\tau^{(n)}(t)\} = s^{2n} J(q) - \sum_{\zeta=0}^{n-1} q^{(n-\zeta-1)} d_\tau^{(\zeta)}(0), \quad n = 1, 2, 3, \dots$$

6- GFI Integral Transform [1]

Nbila G. and Ali F. introduced a new integral transform called the GFI integral transform. This transformation is characterized by the function $h(t)$ as follows:

$$\mathcal{Gf}_h\{h(t)\} = q^3 \int_0^\infty e^{-\frac{t}{q}} h(t) dt = h(q), \quad q > 0$$

GFI Transform of some elementary functions are:

$$1) \mathcal{Gf}_h\{t^n\} = q^5, \quad n \in \mathbb{N}$$

$$2) \mathcal{Gf}_h\{e^{bt}\} = \frac{q^3}{1 + bq}, \quad b \in \mathbb{R}$$

$$3) \mathcal{Gf}_h\{\sin \sin bt\} = \frac{bq^5}{1 + b^2 q^2}$$

$$4) \mathcal{Gf}_h\{\cos \cos bt\} = \frac{q^4}{1 + b^2 q^2}$$

$\mathcal{Gf}_h$ Transform of Derivatives:

Let $\mathcal{Gf}_h\{h(t)\} = h(q)$, then

$$i) \mathcal{Gf}_h\{h'(t)\} = \frac{1}{q} h(q) - q^3 h(0)$$

$$ii) \mathcal{Gf}_h\{h''(t)\} = \frac{1}{q^2} h(q) - q^2 h(0) - q^3 h'(0)$$

$$iii) \mathcal{Gf}_h\{h^{(n)}(t)\} = \frac{1}{q^n} h(q) - q^3 \sum_{j=0}^{n-1} \frac{1}{q^{n-1-j}} h^{(j)}(0), \quad n = 1, 2, 3, \dots$$



Conclusions

Integral transformations are considered paramount to the mathematics community and are researched by scientists in multiple disciplines. Transformations are used to address concerns.

As such, this essay attempts to combine integral transformations in order to present scientists with a single, significant transform that they can utilize to solve equations in a single written passage. This article is intended for this purpose..

Reference

1. Farhat A., Gwila N., GF1integral transform and its some applications, Journal of Alasmarya University: Basic and Applied Sciences, vol. 7, No.2, pp. 84-93, 2022.
2. A N Albukhuttar and I H Jaber, Elzaki transformation of Linear Equation without Subject to any initial conditions, Journal of Advanced Research in Dynamical and control Systems, Vol.11(2), 2019.
3. A N Kathem, On Solutions of Differential Equations by using Laplace Transformation, The Islamic College University Journal, Vol.7(1), 2008.
4. D.V Widde , " The Laplace transform Princeton", University press, USA, Vol.1, 1946.
5. G.K Watugala, "Sumudu transform - a new integral transform to solve differential equations and control engineering problems", Math. Engg. in Indust, vol. 6, 1998.
6. L. Boyadjiev and Y. Luchko, "Mellin integral transform approach to analyze the multidimensional diffusion-wave equations", Chaos, Solitons& Fractals, vol. 102, pp. 127-134, 2017
7. Hussein NA. How to use Temimi Transformation for solving LODE without using any initial Conditions. journal of kerbala university. 2009;7(1):124-8.
8. A. I. Zayed," A convolution and product theorem for the fractional Fourier transform", IEEE Signal processing letters, vol. 5, pp. 101-103, 1998.
9. Rao R. U., Application of ZZ Transform Method for Newton's Law of Cooling, International Journal of Engineering, Science and Mathematics, 6(5), 2017, pp.166-169
10. B Goodwine, "Engineering differential equations: Theory and applications", Springer Science and Business Media , 2010.
11. B. patra, "An Introductionto Integral Transforms", Exporfessor, Bengel Engineering sciencevniversity 8April, 2018.
12. Zaniab . M , Maryame A , Maha .S A Review on integral transforms Bessel's functions International Journal of Engineering and Information Systems (IJEAIS) ISSN: 2643-640X Vol. 8 Issue 10 October - 2024, Pages: 1-4
13. Maha .S and Zaniab . M Review of the Gamma function and Its APplications International Journal of Academic Engineering Research(IJAER) ISSN: 2643- 9085 Vol. 9Issue 9 September - 2025, Pages: 33-36
14. V. Srinivas and C.H. Jayanthi, Application of the new Integral "J-transform" in cryptography, International Journal of Emering Technologies, 11 (2), 2020, pp. 678-682.
15. Eman A. Mansour, Emad A. Kuffi, Sadiq A. Mehdi, The new integral transform "SEE transform " and its Applications, Periodicals of Engineering and Natural Sciences, Vol.9, N0.2, 2021, pp.1016-1029.
16. Alwan Z. M., ZMA-transform method, Journal of interdisciplinary Mathematics, (2021),
17. Mousa A., AMJ Integral transform method: for sol equations , JAMCS, No. JAMCS.70014, (2021).



18. B. patra, "An Introduction to Integral Transforms", Exporfessor, Bengel Engineering sciencevniversity 8April, 2018.
19. Sania Qureshi and Prem Kumar: using shehu integral transform to solve fractional order caputo type initial value problems, Vol.18(2),2019.
20. X J Yang. A new integral transform method for solving steady heat-transfer problem, Ther-mal Science, Vol.20 (2), 2016.