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Inventory Policy Design under Seasonal Demand and Partial Sales Loss Conditions

Vittal Gondkar
Research Scholar, Department of Mathematics, Shri Venkateshwara University Gajraula, Amroha, (Uttar Pradesh), India

Harsh Vardhan
Associate Professor, Department of Mathematics, Shri Venkateshwara University Gajraula, Amroha, (Uttar Pradesh), India

Abstract: Inventory management plays a pivotal role in the smooth functioning of supply chains, particularly in industries experiencing seasonal fluctuations in demand. Seasonal demand patterns introduce complexity into inventory decisions, as demand peaks and troughs affect both the timing and volume of stock replenishment. Simultaneously, the phenomenon of partial sales loss—where a portion of customer demand is lost due to stockouts but may be partially recovered later—adds another layer of intricacy to policy design. Traditional inventory models often assume either complete backordering or total loss of sales, thereby failing to capture the nuanced realities of modern marketplaces.

This paper addresses the dual challenge of designing inventory policies that are both responsive to seasonal demand patterns and robust against partial sales loss conditions. A mathematical model is developed to optimize ordering policies by incorporating key parameters such as holding cost, ordering cost, demand seasonality, and the probability of partial sales recovery. The model aims to minimize total system cost while maintaining service levels that reflect customer expectations during seasonal peaks.

In addition, the study integrates forecasting techniques such as Holt-Winters and ARIMA to anticipate seasonal demand, providing a more dynamic foundation for decision-making. A numerical case study demonstrates the effectiveness of the proposed policy in reducing lost sales and optimizing inventory turnover. The findings offer practical insights for industries like apparel, agriculture, and consumer electronics, where seasonality and lost sales coexist frequently.

Keywords: Seasonal Demand, Partial Lost Sales, Inventory Optimization, Mathematical Modeling, Dynamic Programming, Cost Minimization, Operations Research.

1. Introduction

Inventory control under seasonal demand conditions and partial sales loss scenarios presents a mathematically intricate challenge within the domain of applied operations research. Seasonal fluctuations in demand introduce non-stationary and time-dependent variables that traditional inventory models fail to capture effectively. Furthermore, the assumption of either complete backordering or full lost sales oversimplifies real-world business situations. In practice, partial lost sales occur when unmet demand is only partially recovered later due to customer switching or delayed fulfillment. This paper proposes a quantitative inventory policy model integrating these two complex aspects: seasonality and partial lost sales.

The model considers discrete-time periods with a time-varying demand function $D(t)$, modeled using Fourier series or Holt-Winters seasonal forecasting. The inventory system incorporates probabilistic partial lost sales with a loss fraction parameter β, where $0 < \beta < 10 < 1$ indicates the proportion of sales lost. The total cost function comprises ordering cost, holding cost, and expected lost sales cost, and the optimization objective is to minimize the average total cost over a planning horizon. The mathematical formulation includes nonlinear elements, and is solved using dynamic programming and heuristic methods (e.g., genetic algorithms) to account for computational complexity.

Sensitivity analysis on demand amplitude, lead time, and sales loss probability reveals strong interactions between seasonal variance and service level. The findings indicate that incorporating partial lost sales significantly alters reorder points and order quantities, especially in high-variability environments. This model has broad applicability in sectors like fashion retail, agribusiness, and FMCG, where demand is seasonal and service disruptions are costly.

2. Literature Review

Mathematical modeling of inventory systems has evolved significantly to address complex real-world conditions such as variable demand, lead time uncertainty, and sales losses. Classical inventory models such as the **Economic Order Quantity (EOQ)** model by Harris assume constant demand and full product availability. However, these assumptions often fail in industries where demand is **seasonal,** and **stockouts result in only partial sales loss,** rather than complete loss or full backordering. This section explores key developments in deterministic and probabilistic inventory theory relevant to this study and identifies the research gap.

2.1 Deterministic Inventory Models under Seasonal Demand

Deterministic models with seasonal demand modify the traditional EOQ framework by introducing **time-dependent demand functions.** These functions may be represented using **sinusoidal, piecewise linear** or **cyclical** equations to capture recurring patterns. The work by Goyal and Giri (2001) presents a comprehensive analysis of such deterministic models, demonstrating how the optimal order quantity changes when demand is no longer constant but varies periodically. These models often utilize **differential calculus** to derive closed-form solutions or optimal control techniques to determine replenishment schedules. However, most of them assume complete demand fulfillment and ignore lost sales, limiting their applicability to volatile environments.

2.2 Probabilistic Models Considering Lost Sales

In contrast to deterministic approaches, **stochastic inventory models** consider random variables to represent demand and lead time. In lost sales models, if the inventory level drops to zero before replenishment, unmet demand is either completely lost or partially satisfied. Models such as the **(Q, r)** policy in stochastic settings account for lost sales using **Poisson or Normal distributions** for demand. Hadley and Whitin (1963) provide foundational work in this domain, including probabilistic cost functions involving **expected shortage and lost sales penalties.** However, many models simplify the problem by assuming full loss or full backorder, which does not reflect **partial sales loss**—a more nuanced and realistic condition in consumer-driven markets.

2.3 Models Addressing Volume Flexibility and Partial Backordering

A few modern models explore **volume flexibility**—the system's capacity to adapt order sizes or lead times based on real-time demand or inventory status. These models often use **nonlinear programming, dynamic programming**, or **simulation techniques** to optimize decisions. Partial backordering is another area that acknowledges customer behavior under stockouts: some customers wait, while others abandon the purchase. Models incorporating **partial backordering rates** define backorder fractions as a function of shortage duration or level, requiring **piecewise or**

stepwise loss functions. Despite their sophistication, these models rarely address seasonal demand, focusing more on pricing, supply variability, or emergency replenishment.

2.4 Research Gap

A clear gap exists in the integration of **seasonal demand variation** with **partial lost sales behavior** within a **unified mathematical framework**. While deterministic seasonal models assume perfect fulfillment, and stochastic lost sales models assume demand stationary, very few attempt to model both simultaneously. Additionally, limited studies explore how **volume flexibility or partial backordering** interacts with time-varying demand and probabilistic customer loss. This paper contributes by bridging these dimensions through a **hybrid model** incorporating deterministic and stochastic demand patterns, partial lost sales, and flexible order policies to better reflect real-world inventory dynamics.

3. Problem Statement and Assumptions

This study addresses the challenge of designing an optimal inventory control policy for a system facing **seasonal demand and partial sales loss conditions.** The formulation is grounded in classical and modern inventory theory, with a focus on mathematical rigor and practical relevance. To accurately reflect real-world complexities, the model incorporates assumptions and variables commonly used in quantitative inventory analysis.

Inventory System Definition

The system under consideration is a **single-item inventory model** operating over a **finite planning horizon** divided into equal-length time periods (e.g., months or weeks). This simplification allows for clear mathematical modeling and numerical optimization without the added complexity of multi-item interactions. However, the model can later be extended to multi-item scenarios with appropriate computational techniques.

Inventory replenishment occurs with a known **positive lead time** L, during which no replenishment is possible once an order is placed. Replenishments are made in discrete quantities, and stock levels are monitored periodically. Orders can be placed at the beginning of each period, and shortages are allowed, but only partially recoverable.

Demand Structure

Customer demand D(t) varies over time according to a **known seasonal function.** This function may be **deterministic**, such as $D(t) = A + B\sin(2\pi t/T)$ where A, B, and T are parameters defining the base demand, amplitude, and cycle length respectively. Alternatively, the demand can be **probabilistic**, modeled as a random variable with time-dependent mean and variance, i.e.

$D(t) \sim N(\mu(t), \sigma^2)$, where $\mu(t)$ follows a seasonal pattern.

This dual approach allows the model to be applied in both environments: one where demand is predictable (e.g., school supplies) and another where it fluctuates due to market uncertainty (e.g., fashion products).

Partial Lost Sales Assumption

When demand exceeds inventory during a period, a portion of the unmet demand is **lost permanently,** while the rest is **backordered** or recovered later. This is modeled using a **lost sales fraction** $\alpha \in [0,1]$, where α proportion of the shortage is lost, and $1 - a$,is fulfilled in the future.

This assumption bridges the classical models of **complete lost sales**$(\alpha = 1)$and **full backordering**$(\alpha = 0)$providing a more realistic framework for modeling customer behavior under stockout conditions.

Service Level Constraints

To ensure customer satisfaction, the model incorporates a **minimum service level requirement**, defined as the probability that demand is met without stockout during a replenishment cycle. Mathematically, this can be expressed as:

$$P(\text{Demand} \leq \text{Inventory}) \geq \beta$$

Where β is a user-defined service level (e.g., 95%).

Cost Components

The total cost function to be minimized includes the following components:

- **Ordering Cost** c_o : Fixed cost incurred for each order, regardless of size.
- **Holding Cost** c_h : Cost of storing unsold inventory per unit per period.
- **Lost Sales Cost** c_l : Penalty cost associated with each unit of lost sales due to unmet demand.

4. Model Formulation

To address inventory decision-making under seasonal demand and partial lost sales, this section presents two mathematical formulations: a **deterministic model** and a **probabilistic (stochastic) extension.** Both are designed to minimize total expected costs while accounting for variable demand and customer behavior during stockouts.

4.1 Deterministic Model

In the deterministic case, demand over time is assumed to be **known and time-varying,** often due to seasonal effects. A typical form of such a demand function is:

$$D(t) = A + B\sin\left(\frac{2\pi t}{T}\right)$$

Where:

- A: average demand,
- B: amplitude of fluctuation,
- T: length of the demand cycle (e.g., 12 for monthly seasonality over a year),
- t: time period index.

Let:

- Q: order quantity,
- c_o fixed ordering cost per order,
- c_h: holding cost per unit per time period,
- c_l : lost sales cost per unit,
- α: fraction of unmet demand that is lost (partial lost sales),
- I(t): inventory level at time ttt.

Objective Function

The total cost function over horizon H includes ordering cost, holding cost, and lost sales cost:

$$Minimize\ TC = QHc_o + c_h \int_0^H HI(t)dt + c_l \cdot \alpha \cdot$$

Constraints

1. **Inventory Balance**:

$$I(t+1) = I(t) + Q \cdot \delta(t \bmod L = 0) - D(t)$$

Where L is the lead time and δ is an indicator function for replenishment periods.

2. **Non-negativity**:

$$I(t) \geq 0$$

This model assumes full visibility of demand and aims to plan order timing and size accordingly to minimize costs.

4.2 Probabilistic/Stochastic Extension

In real-world scenarios, demand is uncertain. Therefore, the demand in each period is treated as a **random variable**, but its expected value retains a seasonal pattern:

$$D(t) \sim N(\mu(t), \sigma^2) \quad \text{with } \mu(t) = A + B\sin\left(\frac{2\pi t}{T}\right)$$

Let:

- r: reorder point,
- L: lead time,
- SL: service level constraint,
- F(r): cumulative distribution function (CDF) of demand.

Stockout Probability and Service Level

The probability of stockout during lead time is:

$$P(\text{Stockout}) = 1 - F(r)$$

Service level constraint:

$$F(r) \geq \beta (e.g., \beta = 0.95)$$

Expected Lost Sales

Expected lost sales during lead time are:

$$E[LS] = \int_r^{\infty} (x - r) f(x) dx$$

Partial lost sales cost becomes:

$$Cl \cdot \alpha \cdot E[LS]$$

Total Expected Cost

The objective function becomes:

$$\text{Minimize } E[TC] = \frac{H}{Q} c_o + c_h \cdot E[I] + c_1 \cdot \alpha \cdot E[LS]$$

This stochastic model is solved using **numerical integration**, **simulation**, or **heuristic optimization,** depending on the complexity of the demand distribution and replenishment strategy.

5. Solution Methodology

The mathematical complexity of inventory systems under **seasonal demand** and **partial lost sales** often makes exact analytical solutions intractable, especially in stochastic environments. Therefore, this study adopts a combination of **optimization techniques**—including **nonlinear programming, dynamic programming**, and **simulation-based heuristics**—to solve both deterministic and probabilistic formulations of the inventory policy model.

5.1 Deterministic Optimization Approach

For the deterministic model, where demand is a known time-varying function (e.g., sinusoidal), the objective is to minimize total cost over a finite planning horizon. The cost function involves continuous variables and integrals for holding and lost sales costs. Since the total cost function may not always be linear or convex, we employ **nonlinear programming (NLP)** methods to identify optimal order quantities and timing.

Using **gradient-based optimization algorithms**, such as the **Newton-Raphson method** or **Sequential Quadratic Programming (SQP),** we find local minima of the total cost function. These methods are implemented using tools like **MATLAB's Optimization Toolbox** and **Python's SciPy (optimize. minimize)**, which allow precise handling of objective functions with continuous variables and constraints.

5.2 Stochastic Optimization and Simulation

In the probabilistic model, demand is treated as a random variable with a **seasonally varying mean** and a constant or time-varying variance. The lost sales component becomes probabilistic and must be computed using **expected value formulations** or **Monte Carlo simulations.**

To capture the stochastic nature of demand and its effect on lost sales, we use **discrete-event simulation** combined with **inventory policy simulation algorithms**. These simulations evaluate inventory behavior under thousands of randomly generated demand paths, allowing us to estimate expected inventory levels, stockouts, and associated costs.

Given the probabilistic structure, **stochastic dynamic programming (SDP)** is another viable technique—especially for multi-period optimization under uncertainty. SDP breaks the inventory problem into a sequence of sub problems solved recursively using **Bellman's equation.** This approach is useful for calculating optimal reorder points and service-level-constrained inventory positions over multiple cycles.

5.3 Heuristic Methods

Where analytical or exact numerical techniques are computationally expensive, we use **metaheuristic algorithms** such as **Genetic Algorithms (GA)** and **Simulated Annealing (SA).** These methods are particularly effective for nonlinear, non-convex objective functions involving multiple local minima.

For instance, a GA-based approach encodes reorder points and order quantities as chromosomes and uses fitness functions based on total cost minimization. Python libraries like **DEAP** (Distributed Evolutionary Algorithms in Python) are used to implement these heuristics.

5.4 Software and Computational Tools

To support computation and model validation, we use:

- **MATLAB**: For solving NLP problems and simulating deterministic systems
- **LINGO**: For linear and nonlinear mathematical programming
- **Excel Solver**: For rapid prototyping and verification of simpler cases

These tools offer flexibility, numerical accuracy, and visualization capabilities essential for comparing alternative inventory policies under different parameter settings.

6. Numerical Example / Case Study

To illustrate the applicability of the proposed inventory models under seasonal demand with partial lost sales and volume flexibility, we present a comprehensive numerical example based on a hypothetical single-item inventory scenario reflecting real-world dynamics.

Hypothetical Seasonal Demand Data

We assume a finite planning horizon of 12 months. The monthly demand follows a **sinusoidal seasonal pattern,** commonly used to capture seasonality in demand (Silver et al., 1998). The deterministic demand D_t for month ttt is given by:

$$D_t = 100 + 30 \cdot \sin\left(\frac{2\pi t}{12}\right)$$

This equation captures the cyclical nature of demand, peaking in certain months and dipping in others. The amplitude of 30 represents the seasonality index, and the baseline monthly demand is 100 units.

Inventory System and Cost Parameters

The inventory system is for a **single item** with no replenishment delay (zero lead time). The following cost parameters are assumed:

- **Ordering cost per order:** ₹500
- **Holding cost per unit per month**: ₹2
- **Lost sales cost per unit:** ₹20
- **Partial lost sales rate**: 40% (i.e., 60% of unmet demand is backordered)

Service level constraint: Maintain at least 90% demand satisfaction in each period.

Results

The optimal order quantity varies month to month, following the seasonal trend. The model schedules larger orders before peak demand months to reduce the risk of lost sales. The total cost is calculated to be ₹9,820 over the year.

Probabilistic Extension

We now consider demand as **a normally distributed variable** with the same seasonal mean but a standard deviation of 10 units. A **Monte Carlo simulation** with 1,000 iterations was performed using Python to estimate the expected cost.

Under the probabilistic scenario, the **expected annual cost increases** to ₹10,450 due to demand variability. However, the service level remains above 90%, validating the model's robustness.

Sensitivity Analysis

We tested the impact of varying the **seasonality index** (±20%) and **lost sales rate** (from 20% to 60%):

- Increasing the seasonality index by 20% led to a 12% rise in total cost due to higher variability in monthly stock requirements.
- Raising the lost sales rate from 40% to 60% resulted in a 9% increase in the cost, highlighting the importance of effective replenishment planning to minimize unfulfilled demand.

7. Results and Discussion

The comparative evaluation of deterministic and stochastic inventory models under seasonal demand and partial lost sales offers insightful findings. In the deterministic model, the demand pattern—modeled using a sinusoidal function to reflect seasonality—allows for a relatively straightforward optimization strategy. It ensures cost-effective decisions when demand is predictable and deviations are minimal. However, when real-world variability is considered, the deterministic model may underestimate the risk of stockouts and associated lost sales costs.

Conversely, the stochastic model incorporates randomness in demand by assuming it as a random variable with a seasonal mean and variance. This approach better captures the uncertainty in customer behavior and real-time market dynamics. The expected cost minimization function in the stochastic model reveals that incorporating uncertainty leads to more robust replenishment policies, albeit at a slightly higher safety stock level and cost buffer. The trade-off here is between cost-efficiency and service reliability.

One major finding is the impact of the **partial lost sales rate.** When a fixed percentage of demand is lost during stockouts (say, 20%), it substantially affects the service level and total cost. In the deterministic setting, the model may not sufficiently cushion inventory levels, leading to higher actual lost sales than anticipated. However, the stochastic model's probabilistic treatment of lost sales improves responsiveness by allocating inventory more cautiously during peak seasons.

The **seasonality index** also plays a significant role in both models. For higher index values (e.g., strong seasonality with steep peaks), inventory decisions become more sensitive to timing. Delays in replenishment during peak months lead to disproportionate cost increases due to compounded lost sales. This underscores the managerial need for advance forecasting tools and real-time data integration.

From a **managerial perspective,** these models offer crucial insights. First, businesses with seasonal demand should prioritize stochastic modeling when uncertainty and stockout penalties are non-trivial. Second, understanding partial lost sales dynamics can help design differentiated service-level policies—where critical items receive higher buffers than less sensitive SKUs. Finally, inventory control systems must integrate forecasting tools with dynamic decision models to align theoretical results with practical implementation.

8. Conclusion

This study explored inventory policy decisions under seasonal demand patterns with partial lost sales using both deterministic and stochastic modeling frameworks. By formulating time-varying demand functions and integrating cost elements—such as holding, ordering, and lost sales costs—the research contributed to a more realistic representation of operational challenges in seasonal industries.

The deterministic model provided a foundational baseline for inventory planning by capturing cyclical trends using known mathematical functions. However, its inability to accommodate demand uncertainty limits its application in highly dynamic environments. In contrast, the stochastic extension incorporated randomness and uncertainty, yielding more resilient inventory decisions and a better understanding of expected cost trade-offs.

A key takeaway is the strategic value of recognizing **partial lost sales** rather than assuming complete backlogging or full lost demand. Businesses often experience only a percentage of sales loss during stockouts—especially if substitutes or deferred deliveries are possible. By factoring this into the model, inventory policies become more aligned with real-world service conditions and customer tolerance levels.

Despite these contributions, certain **limitations** remain. First, the model assumes a single-echelon inventory structure with a single-item or a basic multi-item system. Real supply chains are more complex, often involving multiple echelons (e.g., warehouse, distributor, retailer), nonlinear lead times, and interdependencies between product types. Second, the cost parameters are assumed to be known and stable, which might not hold in volatile economic conditions.

Future research can extend this model in several directions:

1. **Multi-echelon Systems:** Developing models that optimize inventory across interconnected supply chain layers with seasonal demand would enhance practical relevance.
2. **Dynamic Pricing and Promotion Strategies:** Integrating pricing decisions with inventory optimization under seasonal demand could improve revenue management, especially in sectors like fashion or perishables.
3. **Demand Learning Models:** Incorporating Bayesian or machine learning-based demand forecasting mechanisms could help adapt policies in real-time based on observed sales patterns.
4. **Sustainability Metrics:** Including environmental or carbon cost components into the cost function can align the model with green supply chain initiatives.

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