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A Method for Ranking Data Envelopment Analysis Based on the Degree of Agreement

Rafid Subhi Hatem Alazzawi
Wasit Governorate Education Directorate

Abstract: One of the best methods of ranking the decision-making units (DMUs) is through cross-efficiency analysis in data envelopment analysis. Nevertheless, the various cross-efficiency evaluation models based on different views provide difference cross-efficiency rankings. The data that come out of the different point of views can be useful and cannot be disregarded. In this paper we are espousing a new composite approach towards ranking DMUs through verifying the Shannon entropy of the cross-efficiency scores that will be acquired through the lens of both perspectives of satisfaction and consensus. We also embrace grey incidence analysis to make a comparative evaluation of ranking of various cross-efficiency models. The computational process of applying Shannon entropy and grey incidence analysis is depicted on an illustration to provide the composite ranking outcome and compare to any other cross-efficiency.

Keywords: efficiency, infeasibility, cross-efficiency.

1. Introduction

Cross-Efficiency Evaluation (CEE) is an advanced method developed to enhance the accuracy of traditional Data Envelopment Analysis (DEA). It improves the reliability of efficiency measurements by avoiding unrealistic weights and provides a unique ranking for each Decision-Making Unit (DMU) from multiple perspectives [1-5]. Shannon entropy, a key concept in information theory, measures uncertainty in information and supports decision-making. Soleimani and Zarepishe applied Shannon entropy to develop a composite efficiency index based on CEE results, allowing for the ranking of DMUs according to their significance [6-9].

Gray Relational Analysis (GRA), rooted in gray systems theory, is used to evaluate complex systems by comparing diverse data from various sources. It calculates the degree of correlation between data sequences, where higher values indicate stronger similarity or agreement between units. In this context, cross-efficiency assessment benefits from multiple perspectives, making it essential to consider all available information. Cross-efficiency ranking thus serves as a collective evaluation tool that integrates both confirmatory and agreement-based insights.

Accordingly, this research proposes a hybrid model that combines cross-efficiency evaluation with Shannon Entropy. Following Wu and Wang, Shannon entropy is applied to the cross-efficiency values to achieve a more accurate and objective ranking of decision-making units. Additionally, we compare ranking results across different models using the Artificial Diffusion Degree (ADD), derived from Gray Relational Analysis, to measure the level of similarity and agreement between the efficiency rankings [10-14]. Gray incident analysis is a valuable tool for evaluating complex systems with incomplete or uncertain information. The gray agreement score accurately reflects the similarity between ranking results across different models, making it especially useful for comparing efficiency rankings generated by multiple methods and strengthening the credibility of the proposed hybrid model.

Several studies have contributed to this area by ranking units using average cross-efficiency scores to reduce bias, integrating DEA with entropy for more precise prioritization, and applying gray analysis to assess the consistency between model outcomes.

2. Basic Definitions of Cross-Functional Evaluation

Suppose every$DMU_j(j = 1,2,\dots,n)$ It produces output by consuming inputs. Also, assuming the unit is under evaluation, the efficiency score can be measured using the model as follows: $m\, x_{ij}(i = 1,2,\dots,m) s y_{rj}(r = 1,2,\dots,s) DMU_d(d = 1,2,\dots,n)\theta_{dd} CCR$

$$\begin{aligned}
&\max \sum_{r=1}^{s} u_{rd} y_{rj} = \theta_{dd},\\
&s.t. \sum_{i=1}^{m} v_{id} x_{ij} - \sum_{r=1}^{s} u_{rd} y_{rj} \geq 0, \qquad j = 1,2,\dots,n,\\
&\sum_{i=1}^{m} v_{id} x_{ij} = 1,\\
&v_{id} \geq 0 \text{ و } i = 1,2,\dots,m,\\
&u_{rd} \geq 0 \text{ و } r = 1,2,\dots,s,
\end{aligned} \tag{1}$$

where and are the corresponding weights of the outputs and inputs, respectively.$\mathbf{u_{rd}}(\mathbf{r = 1,2,\dots,s})\mathbf{v_{id}}(\mathbf{i = 1,2,\dots,m})$Then we can obtain a set of optimal weights for each .$\mathbf{u^*_{rd}}(\mathbf{r = 1,2,\dots,s})\mathbf{v^*_{id}}(\mathbf{i = 1,2,\dots,m})\mathbf{DMU_d}$Total efficiency score$\boldsymbol{\theta^*_{dd}} = \sum_{r=1}^{s} \mathbf{u^*_{rd} y_{rj}}$ **CCR** For is, that$\mathbf{DMU_d}$It represents the optimal relative efficiency value using self-assessment.

If $\mathbf{DMU_d}$ $\boldsymbol{\theta^*_{dd}} = \mathbf{1}$And all the optimal weights$\mathbf{u^*_{rd}}$ and are strictly positive, then we call it efficient.

We use the corresponding optimal weights of the model to calculate the cross-efficiency scores. The traditional cross-efficiency table for is in the form of table number. The cross-efficiency value of each evaluated by is denoted by and is calculated from the following formula:
$\mathbf{v^*_{id} DMU_d CCRDMU_d(1)DMU_j(j = 1,2,\dots,n\,,j \neq d)(1)DMU_j(j = 1,2,\dots,n\,,j \neq d)DMU_d(d = 1,2,\dots,n)}\boldsymbol{\theta_{dj}}$

$$\theta_{dj} = \frac{\sum_{r=1}^{s} u^*_{rd} y_{rj}}{\sum_{i=1}^{m} v^*_{id} x_{ij}} \qquad d, j = 1,2,\dots,n, d \neq j. \tag{2}$$

Table 1: Cross-functional decision-making unit matrix（$\mathrm{DMU_s}$）

	Rating DMU_d						
Rated DMU_j	1	2	3	....	...	n	Mean
1	θ_{11}	θ_{21}	θ_{31}	...	...	θ_{n1}	$\overline{\theta_1}$
2	θ_{12}	θ_{22}	θ_{32}	...	...	θ_{n2}	$\overline{\theta_2}$
3	θ_{13}	θ_{23}	θ_{33}	...	...	θ_{n3}	$\overline{\theta_3}$
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
n	θ_{1n}	θ_{2n}	θ_{3n}	...	...	θ_{nn}	$\overline{\theta_n}$

The model must be solved for each load in order to calculate cross-efficiency scores for all units.$(1)\mathrm{nDMU_j}$ As a result, each has a cross-efficiency score and an optimal efficiency score.$\mathrm{DMUn-1CCR}$The average cross-functional performance scores, defined by Sexton in 1986 as the overall performance assessed by all, are shown in the last column of the table.$\mathrm{DMU_jDMU(1)}$

$$CE_j = \frac{1}{n}\sum_{d=1}^{n}\theta_{dj} \qquad j = 1,2,\dots,n. \tag{3}$$

3. Calculating Cross-Efficiency Based on Satisfaction

Wu et al. proposed a cross-performance evaluation model based on satisfaction degree, which includes a maximin model and two algorithms. For each set of optimal weights, the model selected, which may not be unique, and can be defined as follows, and is$\boldsymbol{DMU_d}\mathbf{CCR}\boldsymbol{W_d}$Show:

$$\begin{gathered} W_d = \{v_d, u_d \left| \sum_{i=1}^{m} v_d x_{ij} - \sum_{r=1}^{s} u_d y_{rj} \geq 0, \right. \\ \theta_d^* \sum_{i=1}^{m} v_d x_{ij} - \sum_{r=1}^{s} u_d y_{rj} = 0, \sum_{i=1}^{m} v_d x_{ij} = 1, \forall j, \\ v_{id} \geq 0, i = 1,2,\dots,m, u_{rd} \geq 0, r = 1,2,\dots,s\} \end{gathered} \tag{4}$$

Therefore, the maximum and minimum cross-efficiency for the corresponding can be defined using the set of possible optimal weights as follows:$\boldsymbol{DMU_k DMU_d W_d}$

$$\begin{aligned} E_{dk}(\underline{E_{dk}}) = \ & max(min) \sum_{r=1}^{s} u_d y_{rj}, \\ s.t.\ \ & \theta_d^* \sum_{i=1}^{m} v_{id} x_{ij} - \sum_{r=1}^{s} u_{rd} y_{rj} = 0, \\ & \sum_{i=1}^{m} v_{id} x_{ij} - \sum_{r=1}^{s} u_{rd} y_{rj} \geq 0, \\ & \sum_{i=1}^{m} v_{id} x_{ij} = 1, j = 1,2,\dots,n, \\ & v_{id} \geq 0, i = 1,2,\dots,m, \\ & u_{rd} \geq 0, r = 1,2,\dots,s. \end{aligned} \tag{5}$$

The set of optimal weights based on the model can be rewritten as follows:W_d(5)

$$\begin{aligned} W_d^{trans} = \{(v_d, u_d) | \bar{E}_{dj} \sum_{i=1}^{m} v_d x_{ij} - \sum_{r=1}^{s} u_d y_{rj} - s_{dj}^{+} = 0, \\ \underline{E}_{dj} \sum_{i=1}^{m} v_d x_{ij} - \sum_{r=1}^{s} u_d y_{rj} + s_{dj}^{-} = 0, \\ \theta_d^{*} \sum_{i=1}^{m} v_d x_{ij} - \sum_{r=1}^{s} u_d y_{rj} = 0, \quad \sum_{i=1}^{m} v_d x_{ij} = 1 \\ v_{id} \geq 0, i = 1,2,\dots,m, \quad u_{rd} \geq 0 \;\; r = 1,2,\dots,s, \\ s_{dj}^{+} \geq 0, s_{dj}^{-} \geq 0, j = 1,2,\dots,n. \} \end{aligned} \tag{6}$$

When trying to choose an optimal set of weights, each one prefers its cross-efficiency to be close to and far from$DMU_d W_d^{trans} DMU_j \bar{E}_{dj}$ $\underline{E}_{dj}$Based on this observation, Wu et al. defined the satisfaction degree for the set of optimal weights selected from as follows:$DMU_j DMU_d W_d^{trans}$

$$\varphi_{dj} = \frac{\sum_{r=1}^{s} u_d y_{rj} / \sum_{i=1}^{m} v_d x_{ij} - \underline{E}_{dj}}{\bar{E}_{dj} - \underline{E}_{dj}}, \qquad \bar{E}_{dj} \neq \underline{E}_{dj}, \forall j. \tag{7}$$

It is observed that when$\varphi_{dj} \epsilon [0,1]$ $\varphi_{dj} = 1$, the new optimal weight set of DMU_dThey produce.$\bar{E}_{dj}$. It should be noted that$\bar{E}_{dj} = \underline{E}_{dj}$ It indicates that Cross-functionalityDMU_j Related toDMU_d.*Based on the optimistic view, Wu et al. introduced a model to select an optimal weight for:*(8)DMU_d.

$$\begin{aligned} \max_{(v_d, u_d) \bar{E}_{dj} \neq \underline{E}_{dj}} \min \frac{s_{dj}^{-}}{s_{dj}^{+} + s_{dj}^{-}} \\ s.t. \;\; \bar{E}_{dj} \sum_{i=1}^{m} v_d x_{ij} - \sum_{r=1}^{s} u_d y_{rj} - s_{dj}^{+} = 0, \\ \underline{E}_{dj} \sum_{i=1}^{m} v_d x_{ij} - \sum_{r=1}^{s} u_d y_{rj} + s_{dj}^{-} = 0, \\ \theta_d^{*} \sum_{i=1}^{m} v_d x_{ij} - \sum_{r=1}^{s} u_d y_{rj} = 0, \quad \sum_{i=1}^{m} v_d x_{ij} = 1 \\ v_{id} \geq 0, i = 1,2,\dots,m, \quad u_{rd} \geq 0, \qquad r = 1,2,\dots,s, \\ s_{dj}^{+} \geq 0, s_{dj}^{-} \geq 0, \bar{E}_{dj} \neq \underline{E}_{dj}, \;\; j = 1,2,\dots,n. \end{aligned} \tag{8}$$

Assuming that the model is a multi-objective linear programming problem, it can be transformed into a single-objective model as follows:$\Phi_d = \min_{\bar{E}_{dj} \neq \underline{E}_{dj}} \frac{s_{dj}^{-}}{s_{dj}^{+} + s_{dj}^{-}}$(8)

$$\begin{aligned} \max_{(v_d, u_d)} \Phi_d \\ s.t. \;\; \bar{E}_{dj} \sum_{i=1}^{m} v_d x_{ij} - \sum_{r=1}^{s} u_d y_{rj} - s_{dj}^{+} = 0, \end{aligned} \tag{9}$$

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$$\underline{E}_{dj}\sum_{i=1}^{m} v_d x_{ij} - \sum_{r=1}^{s} u_d y_{rj} + s_{dj}^- = 0,$$

$$\theta_d^* \sum_{i=1}^{m} v_d x_{ij} - \sum_{r=1}^{s} u_d y_{rj} = 0, \quad \sum_{i=1}^{m} v_d x_{ij} = 1$$

$$\frac{s_{dj}^-}{s_{dj}^+ + s_{dj}^-} \geq \Phi_d$$

$$v_{id} \geq 0, i = 1,2,\dots,m, \quad u_{rd} \geq 0, \qquad r = 1,2,\dots,s,$$

$$s_{dj}^+ \geq 0, s_{dj}^- \geq 0, \bar{E}_{dj} \neq \underline{E}_{dj}, \quad j = 1,2,\dots,n.$$

Therefore, an optimal set of weights that maximizes the satisfaction of others can be generated for each solution of the model. For each satisfying cross-efficiency, the corresponding can be defined as:$\mathbf{DMUDMU_d(9)DMU_jDMU_d}$

$$E_{dj}^{satis} = \frac{\sum_{r=1}^{s} u_{rd}^* y_{rj}}{\sum_{i=1}^{m} v_{id}^* x_{ij}}, \qquad d, j = 1,2,\dots,n. \tag{10}$$

Then the satisfactory cross-performance score can be obtained using the formula:$\mathbf{DMU_j(11)}$

$$E_j^{satis} = \frac{1}{n}\sum_{d=1}^{n} \frac{\sum_{r=1}^{s} u_{rd}^* y_{rj}}{\sum_{i=1}^{m} v_{id}^* x_{ij}}, \; j = 1,2,\dots,n \tag{11}$$

4. Cross-Efficiency Based on Degree of Agreement

Wang et al. proposed maximum agreement as a secondary objective to solve the problem of weight variation. In other words, the secondary objective maximizes the sum of the differences between the self-rated performance score of the given and the performance scores of the others evaluated by. Therefore, we can formulate the secondary objective model for the evaluation as follows:$\mathbf{DMU_kDMUDMU_kDMU_j}$

$$\begin{array}{ll} \min & \sum_{d=1}^{n} (\theta_d^c - \theta_{dj}), \\ s.t. & \theta_{dj} = \frac{\sum_{r=1}^{s} u_{rd} y_{rj}}{\sum_{i=1}^{m} v_{id} x_{ij}} \leq 1, \\ \square & \theta_{jj} = \theta_j^c, \; j = 1,2,\dots,n, \\ \square & u_{rd} \geq 0, \; r = 1,2,\dots,s, \\ \square & v_{id} \geq 0, \; i = 1,2,\dots,m, \end{array} \tag{12}$$

Here θ_d^candθ_{dj} They show the scores of self-assessment and other assessments, respectively. It is clear that $\theta_d^c - \theta_{dj} \geq 0$, which is a known constant here. Therefore, the objective function of the model isθ_d^c(12)$\max\sum_{d=1}^{n}\theta_{dj}$is equivalent to, which is also $\max\sum_{d=1}^{n}\left(\sum_{r=1}^{s}u_{rd}y_{rj}/\sum_{i=1}^{m}v_{id}x_{ij}\right)$Then we can convert the objective function into a formula:(13).

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$$\max \sum_{d=1}^{n} \theta_{dj} = \sum_{d=1}^{n} \left(\sum_{r=1}^{s} u_{rd} y_{rj} - \sum_{i=1}^{m} v_{id} x_{ij} \right) = \sum_{r=1}^{s} \left(u_{rd} \sum_{d=1}^{n} y_{rj} \right) - \sum_{i=1}^{m} \left(v_{id} \sum_{d=1}^{n} x_{ij} \right) \quad (13)$$

Then the model**(12)**It should be written as follows:

$$\begin{aligned} \max \quad & \sum_{r=1}^{s} \left(u_{rd} \sum_{d=1}^{n} y_{rj} \right) - \sum_{i=1}^{m} \left(v_{id} \sum_{d=1}^{n} x_{ij} \right) \\ \text{s.t.} \quad & \sum_{i=1}^{m} v_{id} x_{ij} = 1,\ j = 1,2,\dots,n, \\ \square \quad & \sum_{r=1}^{s} u_{rd} y_{rj} - \sum_{i=1}^{m} v_{id} x_{ij} \le 0, \\ \square \quad & \sum_{r=1}^{s} u_{rd} y_{rj} - \theta_j^c \sum_{i=1}^{m} v_{id} x_{ij} = 0, \\ \square \quad & u_{rd} \ge 0,\ r = 1,2,\dots,s, \\ \square \quad & v_{id} \ge 0,\ i = 1,2,\dots,m. \end{aligned} \quad (14)$$

Agreement refers to the tendency of individuals in a group assessment to have consistent (similar) opinions on the issues being assessed. We use the conventional vector similarity method to measure the extent of group agreement. The extent of similarity between individual and comprehensive assessments (i.e., the average of cross-performance scores) can be calculated using the formula:$(\lambda_j) DMU_j (15)$

$$\lambda_j = \frac{\sum_{d=1}^{n} \left[(\theta_{dj} - B_d)(CE_j - \bar{A}) \right]}{\sqrt{\sum_{d=1}^{n} (\theta_{dj} - B_d)^2} \times \sqrt{\sum_{d=1}^{n} (CE_j - \bar{A})^2}},\ j = 1,2,\dots,n, \quad (15)$$

where is the average cumulative efficiencies of all . Then we can calculate the weighted average cross-efficiency score as follows:$\bar{A} = \sum_{d=1}^{n} CE_j / n, d = 1,2,\dots,nDMU$

$$\zeta_d = \frac{\sum_{j=1}^{n} \lambda_j \theta_{dj}}{\sum_{j=1}^{n} \theta_{dj}},\ d = 1,2,\dots,n, \quad (16)$$

Here the index weights are$\lambda_j (j = 1,2,\dots,n)$ jM. Weighted cross-performance scores are used for rankings.ζ_d**DMU**.

5. Shannon Entropy Method and Gray Analysis

5.1 Combining cross-efficiency and Shannon entropy

We use the Shannon entropy algorithm summarized by Lee to combine cross-validations based on satisfaction and agreement degree to rank units. Let us assume here, and there is a cross-efficiency model. The matrix formula shows. Therefore, the unit ranking approach using Shannon entropy is written as the following algorithm.$nDMUq(17)E_{n \times q}$

Step 1: Obtain satisfactory cross-efficiency scores and consensus cross-efficiency scores.

Step 2: Calculate the cross-efficiency matrix $\mathbf{E_{n\times q}}$:

$$E = \begin{bmatrix} E_{11} & E_{12} & \cdots & E_{1q} \\ E_{21} & E_{22} & \cdots & E_{2q} \\ E_{31} & E_{32} & \cdots & E_{3q} \\ \cdots & \cdots & \cdots & \cdots \\ E_{n1} & E_{n2} & \cdots & E_{nq} \end{bmatrix} \tag{17}$$

Step 3: Normalize the cross-efficiency matrix as follows:

$$\hat{E} = \frac{E_{jp}}{\sum_{j=1}^{n} E_{jp}},\ j = 1,2,\dots,n, p = 1,2,\dots,q. \tag{18}$$

Step 4: Calculate Shannon entropy for each cross-efficiency model as follows:$\mathbf{H_p}$

$$H_p = -(\ln n)^{-1} \sum_{j=1}^{n} \hat{E}_{jp} \ln \hat{E}_{jp},\ p = 1,2,\dots,q, \tag{19}$$

That here $(\ln n)^{-1}$ is the Shannon entropy constant.

Step 5: Let the degree of uncertainty for each cross-validation performance evaluation model.$\mathbf{D_p = 1 - H_p}$

Step 6: Calculate the importance of the model $\mathbf{C_p}$and calculation$\mathbf{w_p = D_p / \sum_{p=1}^{q} D_p, p = 1,2,\dots,q}$As weighting coefficients of the model $\mathbf{C_p}$.

Step 7: Calculate the overall cross-functional performance evaluation scores$\mathbf{E_j^{C^*} = \sum_{p=1}^{q} w_p E_{jp}, j = 1,2,\dots,n}$A larger value indicates that it is better.$\mathbf{E_j^{C^*} DMU}$

5.2 Comparison Between Cross-Performance Scores Through Gray Occurrence Analysis

The artificial degree of occurrence reflects the degree of similarity between the zigzag lines and$\mathbf{X_i}$ $\mathbf{X_j}$and the degree of proximity. The rate of change and with respect to the same values is an index that describes the close relationship between sequences relatively completely.$\mathbf{X_i X_j}$

Step 1: Define occurrence and construct the index sequence. Suppose that a system agent and its observed values at position , are equal, then$\mathbf{X_i k x_i(k), k = 1,2,\dots,n X_i = (x_i(1), x_i(2), \dots, x_i(n))}$Agent behavioral sequence$\mathbf{X_i}$It is called. and$\mathbf{X_i X_j}$They can be a sequence of cross-efficiency scores from specific cross-efficiency models. Show.

$$\mathbf{X_i = (x_i(1), x_i(2), \dots, x_i(n))} \text{ و } \mathbf{X_j = (x_j(1), x_j(2), \dots, x_j(n))}$$

Step 2: Calculate the starting point

Suppose $\mathbf{D_1}$be the primary operator and $\mathbf{X_i D_1 = (x_i(1)d_1, x_i(2)d_1, \dots, x_i(n)d_1)}$in which

$$\mathbf{x_i(k)d_1 = x_i(k)/x_i(1), x_i(1) \neq 0,\ k = 1,2,\dots,n.}$$

We can calculate the starting point, and which are defined as follows:$\mathbf{X_i' X_j'}$

$$X_i' = (x_i'(1), x_i'(2), \dots, x_i'(n))\,,\ \square X_j' = (x_j'(1), x_j'(2), \dots, x_j'(n)).$$

Step 3: Calculate the starting point. Suppose $\mathbf{D_2}$Operator Let be the starting point which is true in the following relation.

$$,x_i(k)d_2 = x_i(k) - x_i(1), k = 1,2, \dots, nX_iD_2 = (x_i(1)d_2, x_i(2)d_2, \dots, x_i(n)d_2)$$

Then for the full degree of occurrence we can calculate the image of the starting point and as follows:$X_i^0 X_j^0$

$$X_i^0 = (x_i^0(1), x_i^0(2), \dots, x_i^0(n))\text{و}\ X_j^0 = (x_j^0(1), x_j^0(2), \dots, x_j^0(n)).$$

For the relative incidence degree, we need to use the method in step 2 to calculate the initial point image of and as follows:$X_i'^0 X_j'^0$

$$X_i'^0 = (x_i'^0(1), x_i'^0(2), \dots, x_i'^0(n))\ \text{و}\ \square X_j'^0 = (x_j'^0(1), x_j'^0(2), \dots, x_j'^0(n)).$$

Step 4: Calculate the absolute incidence rate and calculate the relative incidence rate. By following the above steps, we can calculate the absolute incidence rate as follows:$\varepsilon_{ij}\gamma_{ij}\varepsilon_{ij}$

$$\varepsilon_{ij} = \frac{1 + |s_i| + |s_j|}{1 + |s_i| + |s_j| + |s_j - s_i|}\ i \le j. \tag{20}$$

Here they can be determined as follows:$|s_i|, |s_j|$

- $|s_j| = \left|\sum_{k=2}^{n-1} x_j^0(k) + \frac{1}{2}x_j^0(n)\right|,$
- $|s_i| = \left|\sum_{k=2}^{n-1} x_i^0(k) + \frac{1}{2}x_i^0(n)\right|,$
- $|s_j - s_i| = \left|\sum_{k=2}^{n-1} (x_j^0(k) - x_i^0(k)) + \frac{1}{2}(x_j^0(n) - x_i^0(n))\right|.$

Similarly, we can calculate the relative incidence γ_{ij} Calculate as follows:

$$\gamma_{ij} = \frac{1 + |s_i'| + |s_j'|}{1 + |s_i'| + |s_j'| + |s_j' - s_i'|},\ i \le j. \tag{21}$$

The components of the relationship can be replaced, as described in step 3.**(19)**

Step 5: Calculate the degree of occurrence of artificial gray. This can be calculated by the formula. Following many researchers, we put.**(20)**$\theta = 0/5$

$$\rho_{ij} = \theta\varepsilon_{ij} + (1 - \theta)\gamma_{ij}. \tag{22}$$

Using the above step, we obtain the gray occurrence matrix as follows, which is an upper triangular matrix. In this matrix, the degree of occurrence of artificial gray is$R$ $.r_{ii} = 1, i = 1,2, \dots, n$

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ \square & r_{22} & \cdots & r_{2n} \\ \square & \square & \cdots & \cdots \\ \square & \square & \square & r_{nn} \end{bmatrix} \quad (23)$$

5.3 An application for passenger airline ranking

We One Example Practical for Composition Scores Efficiency Crossed Agreed and satisfactory on Basis Entropy Shannon Presentation We also use the artificial gray incidence to compare different cross-efficiency ranking models. Wang and Chin provided an example to examine the natural cross-efficiency scores of 14 international passenger airlines. These international passenger airlines are evaluated by 3 inputs and 2 outputs. Table 2 shows the input and output data of 14 along with their efficiency scores. According to the last column of Table 2, we can see that 7 of the units are evaluated as efficient units and we cannot make any further distinction between them. Based on the model, the cross-efficiency matrix is shown in Table 3.**DMUCCRCCR**

6. Results of the combined method

Based on the algorithm shown in Section 5.1, we can calculate the combined cross-efficiency scores of the 2 models based on Shannon entropy.

Step 1: We use two models to calculate the cross-efficiency scores of 14 passenger airlines. The results of the two models introduced in Section 4 are shown in Table 4.

Step 2: The elements of the cross-efficiency matrix are extracted from the third and fourth columns of Table 4.$\mathbf{E_{n\times q}}$

Step 3: The normalized cross-efficiency scores are displayed in the second and third columns of Table 5

Table 2: Data14Passenger airlines International

	Inputs				Outputs	
Airlines	x_1	x_2	x_3	y_1	y_2	Efficiency**CCR**
1	5723	3239	2003	26,677	697	**0/8684**
2	5895	4225	4557	3081	539	**0/3379**
3	24,099	9560	6267	124,055	1266	**0/9475**
4	13,565	7499	3213	64,734	1563	**0/9581**
5	5183	1880	783	23,604	513	**1/0000**
6	19,080	8032	3272	95,011	572	**0/9766**
7	4603	3457	2360	22,112	969	**1/0000**
8	12,097	6779	6474	52,363	2001	**0/8588**
9	6587	3341	3581	26,504	1297	**0/9477**
10	5654	1878	1916	19,277	972	**1/0000**
11	12,559	8098	3310	41,925	3398	**1/0000**
12	5728	2481	2254	27,754	982	**1/0000**
13	4715	1792	2485	31,332	543	**1/0000**
14	22,793	9874	4145	122,528	1404	**1/0000**

Step 4: Shannon entropies for the two models, namely and.$\mathbf{(q=2) H_1 = 0/9877 H_2 = 0/9866}$

Step 5: The degree of uncertainty for the 2 models is equal to and$\mathbf{D_1 = 0/0123\ D_2 = 0/0134}$.

Step 6: Using the above values, we simply find the importance degree i.e. and.$\mathbf{w_1 = 0/4794 w_2 = 0/5206}$

Step 7: The composite cross-efficiency scores are obtained, and their results are shown in the second to last column in Table 5.

It ranks first from the perspective of confidence and is followed by. Similarly, it ranks second from the perspective of agreement and is ranked first. We can see from the overall perspective that it ranks higher than. By combining the results of cross-functional models from similar perspectives, more ranking results can be obtained for the units. $\mathbf{DMU_{13} DMU_{11} DMU_{13} DMU_{11} DMU_{13} DMU_{11}}$

Results and Discussion

This section presents and analyzes the results obtained from the input-output data of 14 international passenger airlines using various DEA-based cross-efficiency models. The analysis focuses on the evaluation of efficiency, cross-efficiency, and the comprehensive ranking of decision-making units (DMUs) by integrating multiple methodologies.

1. Efficiency Scores (CCR Model)

Table 2 displays the input variables (x_1: Number of employees, x_2: Operating expenses, x_3: Fleet size) and output variables (y_1: Revenue Passenger Kilometers, y_2: Number of destinations), along with the CCR efficiency scores. Out of the 14 DMUs, six airlines (DMUs 5, 7, 10, 11, 12, and 13) achieved full efficiency with a score of 1.0000, signifying optimal utilization of resources relative to other units.

DMU 2 recorded the lowest efficiency score (0.3379), indicating substantial inefficiencies in operations. This initial CCR analysis provides a baseline view of technical efficiency.

2. Cross-Efficiency Evaluation

Table 3 presents the cross-efficiency matrix, offering insights from a peer evaluation perspective. This method accounts for the assessment of each DMU by all others, thus refining discriminatory power beyond traditional DEA.

DMUs 13, 11, and 5 emerged as leaders in the cross-efficiency scores, consistently receiving high ratings from peers, reflecting stability and mutual recognition of strong performance. In contrast, DMU 2 was persistently poorly rated, corroborating its inefficiency.

3. Multi-Model Cross-Efficiency Assessment

Table 4 provides results from two cross-efficiency models, evaluated across four criteria:

- ✓ Degree of confidence
- ✓ Cross-validation efficiency
- ✓ Degree of agreement
- ✓ Cross-functional agreement

DMUs 11 and 13 demonstrated outstanding performance across all dimensions, maintaining top scores and consistent rankings. DMU 5 also ranked highly, particularly in the agreement-based metrics, underscoring its robustness across evaluative frameworks.

In contrast, DMUs 2 and 6 consistently scored low, confirming their underperformance and weak peer recognition.

4. Integrated Ranking and Correlation Analysis

4.1 Combined Ranking Based on Aggregated Scores

Table 5 contributes findings through normalized scores (N_1 and N_2), producing the final composite efficiency score EjC∗E_j^{C^*}. This integrated approach allows a unified ranking of DMUs:

- ✓ **Top performers**: DMU 13 (Rank 1), DMU 11 (Rank 2), DMU 12 (Rank 3)

- ✓ **Mid-range performers**: DMUs 7, 5, 14, 4, and 9
- ✓ **Lower performers**: DMUs 10, 8, 3, and 6
- ✓ **Poor performers**: DMU 2 ranked lowest (Rank 14), highlighting consistent inefficiency

These results validate the effectiveness and consistency of combining DEA and cross-efficiency models for a multidimensional performance assessment.

4.2 Comparative Model Ranking and Correlation

4.2.1 Detailed Model-Based Ranking (Table 6)

The combined model rankings from Song and Liu, Fan et al., Wang and Chin, hybrid cross-efficiency, and other metrics (confidence and agreement) are summarized in Table 6. The average score across all models provides a final comparative benchmark for each DMU. DMUs 13, 11, and 12 remain top-ranked across these models, while DMU 2 consistently ranks the lowest.

4.2.2 Visual Comparison of Rankings (Figure 1)

Figure 1 illustrates the consistency in DMU rankings across the different cross-efficiency models. The close clustering of top-performing DMUs reflects strong agreement among methods.

4.2.3 Correlation Analysis (Table 7)

To quantify the agreement between different ranking methods, the Spearman rank correlation coefficient was computed (Table 7). High correlation values (eg, 0.9736 between Song and Liu model and CCR) confirm alignment between the traditional DEA method and extended models.

4.2.4 Insights from Correlation Matrix

- ✓ Song and Liu model shows the strongest correlation with CCR and average ranking.
- ✓ Cross-functional agreement and hybrid models correlate highly with consensus ranking, highlighting their effectiveness.

4.2.5 Implications of Ranking Consistency

The consistency in rankings across models strengthens the credibility of the results. It demonstrates that regardless of methodological variation, the core conclusions about DMU performance remain stable.

4.2.6 Final Observations

Models such as Wang and Chin and the hybrid approach contribute significantly to ranking reliability. Their alignment with CCR efficiency confirms their utility in practical decision-making contexts.

Table 3: Cross-functional efficiency matrix for 14 decision-making units

	GoalDMU														Average cross-	
DMU	1	2	3	4	5	6	7	8	9	10	11	12	13	14	efficiency	Rank
1	0/8684	0/4501	0/6225	0/8684	0/8492	0/4726	0/8108	0/7881	0/7031	0/7512	0/8684	0/7713	0/8684	0/8684	0/7543	12
2	0/1719	0/3379	0/0472	0/1719	0/1735	0/0247	0/2479	0/2724	0/2808	0/2058	0/1719	0/2025	0/1719	0/1719	0/1894	14
3	0/8826	0/1942	0/9475	0/8826	0/8844	0/6898	0/7232	0/6833	0/6225	0/7846	0/8826	0/8072	0/8826	0/8826	0/7678	9
4	0/9581	0/4259	0/7034	0/9581	0/9413	0/6973	0/8228	0/7850	0/6991	0/8113	0/9581	0/8341	0/9581	0/9581	0/8222	6
5	0/9653	0/3658	1/000	0/9653	1/000	1/000	0/7704	0/7359	0/7778	1/000	0/9653	1/000	0/9653	0/9653	0/8912	3
6	0/8818	0/1108	0/9563	0/8818	0/8780	0/9766	0/6615	0/6084	0/5099	0/7176	0/8818	0/7478	0/8818	0/8818	0/7554	11
7	0/9211	0/7781	0/4773	0/9211	0/8795	0/3382	1/000	1/000	0/8395	0/7808	0/9211	0/8012	0/9211	0/9211	0/8214	7
8	0/7813	0/6114	0/5162	0/7813	0/7703	0/2924	0/8458	0/8588	0/8208	0/7532	0/7813	0/7613	0/7813	0/7813	0/7242	13
9	0/7855	0/7278	0/5076	0/7855	0/7889	0/2677	0/8782	0/9072	0/9477	0/8375	0/7855	0/8369	0/7855	0/7855	0/7590	10
10	0/7821	0/6354	0/6520	0/7821	0/8250	0/3564	0/7780	0/7944	1/000	1/000	0/7821	0/9719	0/7821	0/7821	0/7803	8
11	1/000	1/000	0/4287	1/000	1/000	0/4418	1/000	1/000	1/000	1/000	1/000	1/000	1/000	1/000	0/9193	1
12	0/9462	0/6336	0/7500	0/9462	0/9602	0/4395	0/9362	0/9395	0/9998	1/000	0/9462	1/000	0/9462	0/9462	0/8850	4
13	1/000	0/4257	1/000	1/000	1/000	0/4555	1/000	1/000	1/000	0/9843	1/000	1/000	1/000	1/000	0/9190	2
14	1/000	0/2272	1/000	1/000	1/000	1/000	0/7795	0/7275	0/6478	0/8569	1/000	0/8838	1/000	1/000	0/8659	5

Table 4: Results of 2 different cross-efficiency models

(DMU)	Degree of confidence	Cross-validation efficiency	Degree of agreement	Cross-functional agreement
1	0/9062(5)	0/8187(8)	0/7849(2)	0/7884(11)
2	0/6703(14)	0/2339(14)	0/1945(14)	0/1866(14)
3	0/7327(10)	0/7461(12)	0/5450(12)	0/8072(8)
4	0/7900(8)	0/8414(6)	0/7849(3)	0/8585(6)
5	0/6808(11)	0/7976(10)	0/7905(1)	0/9223(3)
6	0/6705(12)	0/6913(13)	0/4141(13)	0/7905(10)
7	0/9780(3)	0/9848(3)	0/7083(9)	0/8481(7)
8	0/9589(4)	0/8345(7)	0/6913(13)	0/7487(13)
9	0/8791(7)	0/8629(5)	0/5848(11)	0/7796(12)
10	0/6703(13)	0/7803(11)	0/7521(8)	0/8036(9)
11	1/0000(1)	1/0000(1)	0/7849(3)	0/9419(2)
12	0/8957(6)	0/9383(4)	0/7698(7)	0/9179(4)
13	1/0000(2)	0/9988(2)	0/7849(3)	0/9626(1)
14	0/7535(9)	0/8096(9)	0/7849(3)	0/9049(5)

Table 5: Combined normalized cross-efficiency scores

(DMU)	N_1	N_2	$E_j^{C^*}$	Rank
1	0/0722	0/0700	0/8029	9
2	0/0206	0/0166	0/2093	14
3	0/0658	0/0717	0/7779	12
4	0/0742	0/0762	0/8503	7
5	0/0703	0/0819	0/8625	5
6	0/0610	0/0702	0/7429	13
7	0/0869	0/0753	0/9137	4
8	0/0736	0/0665	0/7898	11
9	0/0761	0/0692	0/8195	8
10	0/0688	0/0714	0/7924	10
11	0/0882	0/0836	0/9698	2
12	0/0827	0/0815	0/9277	3
13	0/0882	0/0855	11	1
14	0/0714	0/0804	8	6

From Table 6 and Figure 1, it can be concluded that the model using self-assessment gives a higher evaluation result than the cross-efficiency model. The cross-efficiency score obtained from Fan et al.'s model has a good ability for rankings, but the zero cross-efficiency turns it into infinity. The rankings are high for all models, and they always appear in the top 3.

places in the table. In all models, it has the lowest rank. Table 7 shows the Spearman correlation coefficient among the different ranks. **CCRDMUDMU$_2$DMU$_{11}$DMU$_{13}$DMU$_2$**

*Table 6: Ranking results**DMU***

DMU	CCR	Average	Efficiency Crossed Confidence	Cross-functional agreement	Hybrid cross-efficiency	Wang and Chin model	Fan et al.'s model	Song and Liu model
1	0/8684(12)	0/7543(12)	0/8187(8)	0/7884(11)	0/8029(9)	0/7049(11)	0/4885(13)	0/7095(12)
2	0/3379(14)	0/1894(14)	0/2339(14)	0/1866(14)	0/2093(14)	0/1912(14)	0/0000(14)	0/1812(14)
3	0/9475(11)	0/7677(9)	0/7461(12)	0/8072(8)	0/779(12)	0/7154(10)	0/5821(11)	0/7334(10)
4	0/9581(9)	0/8222(6)	0/8414(6)	0/8585(6)	0/8503(7)	0/7733(7)	0/6528(7)	0/7836(6)
5	1/000(1)	0/8912(3)	0/7976(10)	0/9223(3)	0/8625(5)	0/8764(2)	0/8781(3)	0/8714(1)

6	0/9766(8)	0/7554(11)	0/6913(13)	0/7905(10)	0/7429(13)	0/7024(12)	0/6340(8)	0/7392(8)
7	1/000(1)	0/8214(7)	0/9848(3)	0/8481(7)	0/9137(4)	0/7711(8)	0/7070(6)	0/7693(7)
8	0/8588(13)	0/7242(13)	0/8345(7)	0/7487(13)	0/7898(11)	0/6906(13)	0/5000(12)	0/6776(13)
9	0/9477(10)	0/7590(10)	0/8629(5)	0/7796(12)	0/8195(8)	0/7378(9)	0/5880(10)	0/7120(11)
10	1/000(1)	0/7803(8)	0/7803(11)	0/8036(9)	0/7924(10)	0/7813(6)	0/6066(9)	0/7368(9)
11	1/000(1)	0/9193(1)	1/000(1)	0/9419(2)	0/9698(2)	0/9041(1)	0/9740(1)	0/8703(2)
12	1/000(1)	0/8850(4)	0/9383(4)	0/9179(4)	0/9277(3)	0/8541(4)	0/8136(5)	0/8323(5)
13	1/000(1)	0/9190(2)	0/9988(2)	0/9626(1)	0/9805(1)	0/8723(3)	0/8973(2)	0/8582(3)
14	1/000(1)	0/8659(5)	0/8096(9)	0/9049(5)	0/8592(6)	0/8140(5)	0/8394(4)	0/8413(4)

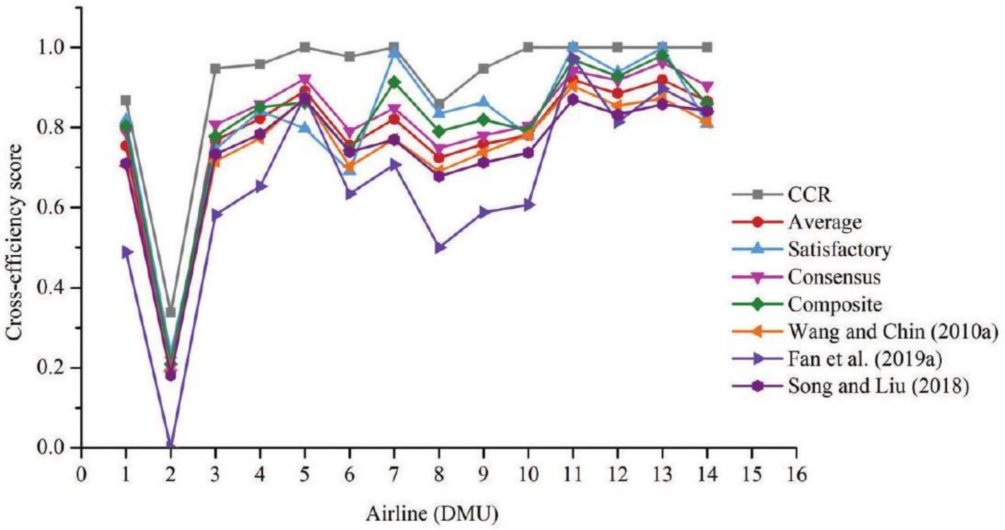

Figure 1: Comparison of rankings of different cross-efficiency models

Table 7: Spearman rank correlation coefficient between different rankings

Models	CCR	Average	Efficiency Crossed Confidence	Cross-functional agreement	Hybrid cross-efficiency	Wang and Chin model	Fan et al.'s model	
CCR	1/000							
Average	0/8566	1/000						
Cross-validation efficiency	0/4483	0/6044	1/000					
Cross-functional agreement	0/8238	0/9780	0/5473	1/000				
Hybrid cross-efficiency	0/7628	0/8857	0/8549	0/8549	1/000			
Wang and Chin model	0/8660	0/9736	0/5648	0/9253	0/8637	1/000		
Fan et al.'s model	0/8848	0/9560	0/5824	0/9385	0/8549	0/9121	1/000	
Song and Liu model	0/8613	0/9560	0/4593	0/9560	0/8022	0/9209	0/9736	1/000

There is a high correlation between the results of different rankings (Spearman's rank correlation coefficients are greater than 0.8). Therefore, correlation information between the results of different rankings is rarely available. In the next step, we calculate the degree of occurrence of artificial gray among different cross-efficiency scores, which can be viewed as the index sequence resulting from different cross-efficiency models. Here, following the algorithm given in Section

5.2, we illustrate the calculation of the degree of occurrence of artificial gray among the sequences of cross-efficiency confidence and agreement by an example.

Step 1: Define the occurrence and formation of the index sequence. SupposeX_iIndicatesCross-efficiency sequence of confidence andX_jLet denote the sequence of cross-agreement efficiency. Then we can obtain two sequences with the same time interval as follows:

$$X_i = (0/8187,0/2339,0/7461,0/8414,0/7976,0/6913,0/9848,$$
$$0/8345,0/8629,0/7803,1/0000,0/9383,0/9988,0/8096),$$
$$X_j = (0/7884,0/1866,0/8072,0/8585,0/9223,0/7905,0/8481,$$
$$0/7487,0/7796,0/8036,0/9419,0/9179,0/9626,0/9049).$$

Step 2: Calculate the initial image.

$$X_i' = (1/0000,0/2857,0/9113,1/0277,0/9742,0/8444,1/2029,$$
$$1/0193,1/0540,0/9531,1/2214,1/1461,1/2200,0/9889),$$
$$X_j' = (1/0000,0/2367,1/0238,1/0889,1/1698,1/0027,1/0757,$$
$$0/9496,0/9888,1/0193,1/1947,1/1643,1/2210,1/1478).$$

Step 3: Calculate the starting point image. For the full occurrence rate, we can calculate the starting point image and$X_i^0 X_j^0$Calculate as follows.

$$X_i^0 = (0/0000,-0/5848,-0/0726,0/0227,-0/0211,-0/1274,0/1661,$$
$$0/0158,0/0442,-0/0384,0/1813,0/1196,0/1801,-0/0091),$$
$$X_j^0 = (0/0000,-0/6018,0/0188,0/0701,0/1339,0/0021,0/0597,$$
$$-0/0397,-0/0088,0/0152,0/1535,0/1295,0/1742,0/1165).$$

For our relative occurrence degree, we use the second step to calculate the starting point image and we use it as follows.$X_i'^0 X_j'^0$

$$X_i'^0 = (0/0000,-0/7143,-0/0887,0/0277,-0/0258,-0/1556,0/2029,$$
$$0/0193,0/0540,-0/0469,0/2214,0/1461,0/2200,-0/0111),$$
$$X_j'^0 = (0/0000,-0/7633,0/0238,0/0889,0/1698,0/0027,0/0757,$$
$$-0/0504,-0/0112,0/0193,0/1947,0/1643,0/2210,0/1478).$$

Step 4: Calculate the degree of complete occurrence$\varepsilon_{ij}$and the relative degree of occurrence.γ_{ij}

We can obtain the absolute degree of occurrence. Similarly, and the relative degree of occurrence.

$|s_i| = 0/1191, |s_j| = 0/1649, |s_j - s_i| = 0/0459 \varepsilon_{ij} = 0/9655 |s_i| = 0/1454, |s_j| = 0/2092, |s_j - s_i| = 0/0638 \gamma_{ij} = 0.9551$

Step 5: Calculate the degree of occurrence of artificial gray. We obtain. We repeat the above step and then obtain the gray occurrence matrix shown in Table 8.$\rho_{ij} = \theta\varepsilon_{ij} + (1-\theta)\gamma_{ij} = 0/9603$

From Table 8, we can conclude that the degree of occurrence of artificial gray is between the confidence cross-efficiency and the agreement cross-efficiency, which is the maximum value. Such a high number indicates that the two sequences are very similar to each other. Therefore, combining these two views in a combined ranking method via Shannon entropy is of great importance.**0/9603**. The relative gray occurrence degree between confidence and composite cross-efficiency is equal to. While the artificial gray occurrence degree between agreement and composite cross-efficiency is equal to. Therefore, compared to the agreement cross-efficiency, the ratings associated with cross-efficiency are closer to the composite cross-efficiency.**0/92190/8884**. From the perspective of cross-efficiency, the average degree of occurrence of artificial gray between it and the cross-efficiency of agreement is equal to**0/9424**Which is the biggest.The next largest is equal to**0/9413**It is thatThe degree of occurrence of artificial gray is between the mean and the cross-efficiency of the Song and Liu model. The conventional vector similarity method to calculate the degree.

Table 8: Gray occurrence matrix for different rankings

Models	CCR	Average	Efficiency Crossed Confidence	Cross-functional agreement	Hybrid cross-efficiency	Wang and Chin model	Fan et al.'s model	Song and Liu model
CCR	1/000	0/8187	0/7588	0/7812	0/7185	0/9340	0/6757	0/8604
Average		1/000	0/9073	0/9424	0/8438	0/8669	0/6114	0/9413
Cross-validation efficiency			1/000	0/9603	0/9219	0/7993	0/5914	0/8598
Cross-functional agreement				1/000	0/884	0/8249	0/5990	0/8906
Hybrid cross-efficiency					1/000	0/7528	0/5774	0/8038
Wang and Chin model						1/000	0/6505	0/9155
Fan et al.'s model							1/000	0/6256
Song and Liu model								1/000

Agreement is used and the coefficient of variation method introduced by Song and Liu is used to assign weights based on the degree of dispersion of the data. Therefore, as expected, these two ranking methods are more like the mean cross-efficiency.

The degree of occurrence of artificial gray between Wang and Chin's ranking and the model**CCR**is equal to. Then the value**0/93400/9155**It appears that the degree of occurrence of artificial gray is between the Wang and Chin and Song and Liu rankings. The similarity between Fan et al's rankings and other models is not very high. To summarize, the cross-validation results obtained from our combined view of confidence and agreement based on Shannon entropy should be more widely adopted in practical decision-making.

Conclusion

Cross-validation is an efficient method that is widely used for ranking. DMU is used. This paper proposes a method to combine cross-efficiency scores from the perspectives of confidence and agreement based on Shannon entropy. A comparison of the rankings of different cross-efficiency models with different perspectives based on the degree of occurrence of artificial gray is carried out. The main result of this study is as follows:

The ranking between the cross-performances of agreement and confidence has the highest similarity. In the comparison of agreement cross-efficiency, the confidence cross-efficiency ranking is closer to the results of the composite cross-efficiency ranking. The cross-efficiency results obtained from the combined perspective of consensus and confidence based on Shannon entropy should be more widely adopted in practical decision-making. Our method represents a prominent and practical solution for combining different perspectives used in previous cross-efficiency models, which have led to different results.

However, there are some limitations in this paper. For the results of Fan et al. and Song and Liu, we use the optimistic cross-efficiency model to solve the problem of non-uniqueness of solutions. In addition, the sample size used in the practical example is small compared to the number of inputs and outputs, which may affect our results. The evaluation of cross-efficiency in variable returns to scale is also an important issue for future studies. In the future, we can consider the secondary objective from a special perspective to improve our combined method. In addition, following the general idea of this study, the combination of data envelopment analysis and multi-criteria decision-making techniques such as (analytic hierarchy process), (ranking technique based on similarity to the ideal solution) and gray event data analysis can be applied to practical problems.

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