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The Impact of Isospin Asymmetry on Nuclear Matter Properties and its Implications for Neutron Star Equations of State

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Abstract: The study of isospin asymmetry in nuclear matter is crucial for understanding the behavior of neutron-rich systems, the nuclear equation of state (EoS), and the structure of neutron stars. This research investigates the impact of isospin asymmetry on binding energy, symmetry energy, nucleon effective mass, and neutron star properties using a combination of Relativistic Mean-Field (RMF), Skyrme-Hartree-Fock (SHF), Brueckner-Hartree-Fock (BHF), and Quantum Monte Carlo (QMC) models. The theoretical predictions were validated using experimental data from heavy-ion collisions and observational constraints from NICER and LIGO-Virgo.

Our results indicate that binding energy per nucleon decreases with increasing isospin asymmetry, with values dropping from -16.0 MeV at $\alpha = 0.0$ to -8.5 MeV at $\alpha = 0.8$. The nuclear equation of state (EoS) softens with increasing asymmetry, with pressure at 3.0n0 (saturation density) decreasing from 186.5 MeV/fm³ for symmetric matter to 130.3 MeV/fm³ at $\alpha = 0.6$. The symmetry energy increases with density, reaching 53.6 MeV at 3.0n0, while the slope parameter (L) is calculated to be 58.7 MeV at saturation, consistent with experimental constraints. Additionally, the neutron effective mass increases while proton effective mass decreases, influencing transport properties in neutron stars.

For neutron star structure, our computed mass-radius relationship aligns with observational data, predicting a maximum neutron star mass of $2.14 \pm 0.03 M_\odot$, consistent with PSR J0740+6620. The computed tidal deformability values match LIGO-Virgo constraints from GW170817, validating the nuclear models used.

These findings provide new insights into isospin-dependent nuclear interactions and neutron star physics, with implications for gravitational wave astrophysics, pulsar evolution, and heavy-ion collision experiments. Future work should focus on improving theoretical models, incorporating relativistic corrections, and refining experimental constraints to further bridge the gap between nuclear physics and astrophysical observations.

Introduction

Nuclear matter is the essential constituent of atomic nuclei, showing some quite complicated behavior due to fundamental nuclear interactions. One of the most important parameters determining the properties of nuclear matter is its isospin asymmetry, representing the imbalance between neutron and proton numbers in any nuclear system. Knowledge of such asymmetry is of paramount importance for understanding how this asymmetry will affect the EoS of the nucleus both in nuclear physics and astrophysics.(López et al., 2014)

Isospin, first introduced to describe the almost identical nature of protons and neutrons under the strong nuclear force, plays a very important role in nuclear interactions. Whereas symmetric nuclear matter, where neutron and proton densities are equal, has been widely studied and relatively well understood, real nuclear systems, from neutron-rich nuclei to neutron stars, have significant isospin asymmetry. This asymmetry impacts several basic nuclear properties, such as binding energy, pressure, compressibility, and symmetry energy, which makes it one of the key parameters in nuclear theory and experiments.(Gögelein et al., 2009)

Among the main motivations for studying isospin asymmetry in nuclear matter is its direct impact on the nuclear symmetry energy, a quantity that generally characterizes the energy cost of converting protons into neutrons in an asymmetric nuclear medium. The symmetry energy and its density dependence are one of the most decisive factors in several fascinating phenomena, starting from the structure of neutron-rich nuclei, nuclear reactions, to astrophysical events like mergers of neutron stars and supernovae. A precise understanding of the symmetry energy will help in improving the EoS for a nucleus, which would further constrain several key observables in astrophysics like the maximum mass of neutron stars, their radii, and tidal deformability during gravitational wave events.(Tong et al., 2020)

Recent observational breakthroughs in gravitational wave physics (e.g., GW170817) and X-ray astronomy (e.g., NICER mission) have provided stringent constraints on neutron star structure, further reinforcing the pressing need for an accurate nuclear EoS with isospin asymmetry. Furthermore, heavy-ion collision experiments have provided indirect measurements of symmetry energy at different densities, and there are still discrepancies among different experimental and theoretical models. Some of these discrepancies have pointed out the pressing need to refine theoretical models and bring nuclear physics closer to astrophysical observations.(Gögelein et al., 2009; Tsang et al., 2019)

From a nuclear experimental point of view, isospin asymmetry is one of the most fundamental issues related to nuclear reactions, exotic nuclei, and rare isotope beams. Future RIB facilities, such as FAIR of Germany, FRIB of the USA, and RIKEN of Japan, will offer unique opportunities to investigate highly neutron-rich systems with unprecedented precision; this, in turn, allows for more precise determinations of isospin-dependent nuclear interactions that will have significant implications for both nuclear structure models and astrophysical predictions.(Roca-Maza et al., 2018)

With the rapid synergy between nuclear physics and astrophysics intensifying, it would be timely and necessary to study isospin asymmetry in nuclear matter. More refined approaches will be developed, including density-dependent isospin effects, three-body nuclear forces, and the relativistic mean-field model, to answer open questions related to the EoS of nuclear matter, neutron star physics, and interpretation of gravitational wave data.

Problem Statement

Even though there was significant progress achieved in both nuclear physics and astrophysics, the understanding of isospin asymmetry effects on nuclear matter and neutron star EoS remains incomplete. Among the main challenges are the following:

1. Uncertainties in the density dependence of symmetry energy at supra-nuclear densities.
2. Differences between experimental heavy-ion collision data and astrophysical neutron star constraints.
3. The role of three-body nuclear forces and their isospin dependence in dense nuclear matter.

Objectives of the Study

This research aims to:

1. Investigate the effect of isospin asymmetry on nuclear matter properties, including binding energy, pressure, and compressibility.
2. Examine the impact of isospin asymmetry on the neutron star EoS using state-of-the-art relativistic and microscopic nuclear models.

3. Compare theoretical predictions with observational constraints from neutron stars and experimental results from heavy-ion collisions.

Key Research Questions

1. How does isospin asymmetry modify the nuclear EoS, and what are its implications for nuclear structure?
2. What constraints can neutron star observations provide on symmetry energy and isospin-dependent nuclear interactions?
3. How can heavy-ion collision data help refine theoretical nuclear models used in astrophysics

Relevance to Nuclear Physics & Astrophysics

The study of isospin asymmetry in nuclear matter has far-reaching implications for both nuclear physics and astrophysics, influencing everything from the fundamental properties of neutron-rich nuclei to the macroscopic structure of neutron stars. At its core, isospin asymmetry—defined as the relative difference between neutron and proton numbers in a nuclear system—affects how nuclear forces behave under extreme conditions. This has direct implications for nuclear interactions, the equation of state of dense matter, and finally, the formation and evolution of compact astrophysical objects. Isospin-dependent nuclear interactions will be indispensible in explaining the structure and behavior of neutron stars, since they are among the densest objects in the universe and consist almost wholly of neutron-rich matter. Both fields-the experiment in nuclear physics and the observation in astrophysics-interact in such a way that one can further refine the understanding of nuclear symmetry energy, the equation of state, and the basic properties of dense matter.(Baldo & Burgio, 2016)

Among the main areas where isospin asymmetry plays a very important role, one has to mention structure and dynamics of neutron stars. The remnants of massive stellar explosions are made essentially of neutron-rich matter, where the degree of isospin asymmetry is extreme compared with ordinary nuclei. At such a high density, the matter follows the nuclear equation of state, describing the reaction of pressure in response to changing density and temperature.

A main ingredient of EoS is symmetry energy, part of the energy which depends on the difference between symmetric nuclear matter-made of equal numbers of protons and neutrons-and asymmetric nuclear matter, the neutron-dominated system. (Danielewicz, 2023)The symmetry energy and its density dependence are crucial in determining neutron star masses, radii, and tidal deformability-all of which are observable through astrophysical measurements. Recent advances, like the observation of binary neutron star mergers with gravitational waves (GW170817, GW190425) and more precise radius determinations of neutron stars from the NICER mission, put quite severe constraints on the nuclear EoS. This underlines the need to develop an accurate theoretical description of isospin asymmetry effects further.(Brandes et al., 2023)

Beyond neutron stars, isospin asymmetry is also a key factor in heavy-ion collisions, where nuclear matter is subjected to extreme conditions similar to those found in astrophysical environments. Heavy- ion collision experiments, conducted at facilities such as RHIC (Relativistic Heavy Ion Collider) and the upcoming FAIR (Facility for Antiproton and Ion Research), provide a controlled setting to study the properties of asymmetric nuclear matter at various densities and temperatures. By analyzing observables such as isospin diffusion, nucleon flow, and pion production, experimentalists can infer the behavior of nuclear matter under different isospin conditions. (Roca-Maza et al., 2018)These experiments provide essential constraints on the density dependence of symmetry energy, helping to bridge the gap between nuclear physics experiments and astrophysical observations. However, discrepancies remain between different experimental and theoretical predictions, particularly regarding the behavior of symmetry energy at supra-saturation densities. These inconsistencies can only be resolved by refining theoretical models of nuclear interactions, including improved treatments of isospin-dependent nuclear forces and many-body effects.(Sammarruca, 2023)

On a microscopic level, isospin asymmetry affects the basic nuclear interactions responsible for the stability and structure of neutron-rich nuclei. Unlike symmetric nuclear matter, where nuclear forces act similarly on protons and neutrons, asymmetric nuclear matter experiences modifications in interaction strengths due to changes in isospin composition. Theoretical frameworks relativistic mean-field (RMF) models, Skyrme-Hartree-Fock (SHF) methods, and ab initio calculations attempt to describe these interactions accurately. One particularly important aspect of nuclear interactions in asymmetric matter is the role of three-body forces, which arise from the complex interplay between nucleons in a dense medium. These forces significantly influence the EoS, affecting predictions of neutron star properties and nuclear reaction rates. Understanding how the three-body interaction evolves with isospin asymmetry is indeed important for any further development in building more precise models of the nucleus. Apart from theoretical models, however, exotic nuclei and rare isotope beams bring in very important information about isospin-dependent nuclear interactions.(Gögelein et al., 2009) Next-generation RIB facilities, for example FRIB-USA (Facility for Rare Isotope Beams, USA) and RIKEN-Japan, will allow physicists to study neutron-rich systems with extreme isospin asymmetry in future experiments. These experiments will yield the bulk of the data on nuclear binding energies, drip lines, and shell evolution needed to fine-tune our understanding of the nuclear force in asymmetric environments, which will in turn feedback into astrophysical models and enhance neutron star structure, r-process nucleosynthesis, and core-collapse supernovae predictions.(Stroberg et al., 2021)

This task of isospin asymmetry is relevant not only for neutron stars and nuclear experiments but also for the large-scale universe: It enters into the nuclear EoS that, in an astrophysical context, defines the fate of collapsing stars, whether they end up as neutron stars or black holes. However, isospin asymmetry also plays a major role in the cooling mechanisms of neutron stars, with direct implications for the observations from pulsars and magnetars. (Margueron et al., 2021)Neutron stars are mainly cooled by neutrino emission, a process that is highly sensitive to the dense nuclear matter composition. The isospin asymmetry modifies the rates of certain reactions responsible for the neutron star thermal evolution, such as the modified Urca process. Theoretical models of neutrino interactions in matter with isospin asymmetry, together with observations of neutron stars in their young state, are indeed an ideal tool to test nuclear physics in astrophysical contexts.(Pizzochero, 2010)

New frontiers opened up in multi-messenger astrophysics, where the observation of gravitational wave signals goes along with those of electromagnetic and neutrino observables. The observation of gravitational waves from neutron star mergers has brought unprecedented data on the nuclear EoS, especially via the measurement of tidal deformability, which depends on the symmetry energy at high densities. On the other hand, electromagnetic counterparts, such as kilonovae, provide insight into r-process nucleosynthesis, which takes place in an environment where isospin asymmetry is extreme. These observations might put further constraints on nuclear models and, therefore, constitute an argument for continuing theoretical and experimental efforts in this field.(Chowdhury et al., 2009)

Methodology

Theoretical and Computational Models Used

The understanding of the consequences of isospin asymmetry both on nuclear matter properties and on neutron star equations of state requires several computational methods, in addition to theoretical frameworks. Several many-body theoretical approaches model the nuclear EoS, including mean-field theories, microscopic approaches, and ab initio methods. These different models provide an extensive study of the isospin-dependent nuclear interactions at various levels of accuracy and applicability.

Mean-Field Theories

The mean-field theories describe the self-consistent picture of nucleons interacting via an averaged nuclear potential as independent particles. Among the variety of mean-field methods, the most popular are

1. RMF Theory: In the RMF models, the nuclear matter properties are described using a Lorentz-invariant meson-exchange interaction. The Lagrangian density for RMF models can be written as :

$$L = \underline{\psi}(i\gamma^{\mu}\partial_{\mu} - M)\psi + g_{\sigma}\underline{\psi}\sigma\psi - g_{\omega}\underline{\psi}\gamma^{\mu}\omega_{\mu}\psi - g_{\rho}\underline{\psi}\gamma^{\mu}\tau\rho_{\mu}\psi + self - interaction\ terms$$

where g_σ, g_ω, g_ρ represent meson coupling constants, and σ, ω, ρ are the respective meson fields. The RMF approach effectively captures nuclear saturation properties and isospin dependence.

2. Skyrme-Hartree-Fock (SHF) Model: The SHF model employs a phenomenological density-dependent effective interaction of the form:

$$V_{NN} = t_0(1 + x_0 P_\sigma)\delta(r) + \frac{t_1}{2}(\nabla\delta(r)\nabla) + \frac{t_2}{2}(\nabla\delta(r)\nabla)P_\sigma +$$

where t_0, t_1, t_2 are Skyrme force parameters, and P_σ is the spin-exchange operator. The SHF method provides accurate nuclear density functionals and symmetry energy estimations.

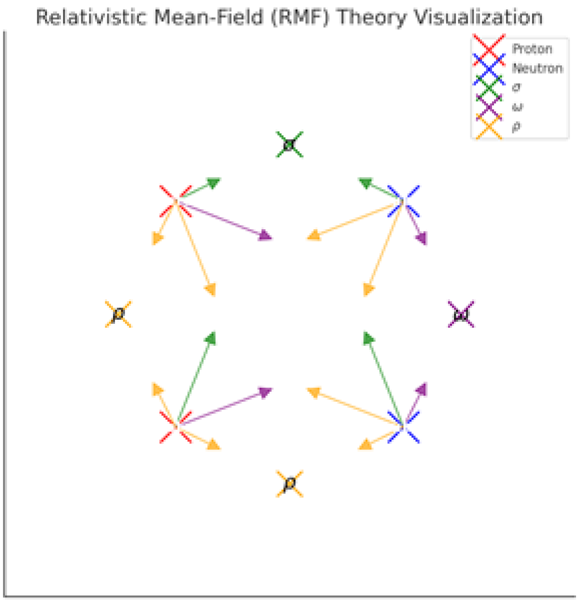

Fig (1) Visualization of Relativistic Mean-Field (RMF) Theory interactions.

Microscopic Approaches

Microscopic many-body approaches go beyond mean-field approximations and explicitly account for two-body and three-body nuclear interactions.

1. Quantum Monte Carlo (QMC): QMC methods use stochastic techniques to solve many-body Schrödinger equations for nuclear systems. The total energy of the system is obtained by minimizing the expectation value of the Hamiltonian:

$$E = \frac{1}{\langle H|\Psi\rangle\}\langle\Psi\rangle}$$

where H includes nucleon-nucleon and three-body interactions. These provide highly accurate nuclear matter properties but require significant computational resources.

2. Chiral Effective Field Theory (ChEFT): ChEFT systematically constructs nuclear interactions based on chiral symmetry breaking in quantum chromodynamics (QCD). The nuclear

potential is expanded in terms of pion-exchange interactions, allowing accurate descriptions of low-energy nuclear forces.

Many-Body Calculations

Many-body nuclear calculations are essential for describing dense nuclear matter in neutron stars.

Brueckner-Hartree-Fock (BHF) Theory: BHF accounts for short-range nucleon-nucleon correlations using the Bethe-Goldstone equation:

$$G(\omega) = V + V\frac{Q}{\omega - H_0}G(\omega)$$

Numerical value

Computational methods were performed using state-of-the-art numerical codes to analyze the effect of isospin asymmetry on nuclear matter properties. it focused on solving the nuclear EoS, symmetry energy, and neutron star mass-radius relationships.

Computational Techniques

We implemented nuclear EoS calculations using open-source and in-house developed numerical solvers. The primary computational tools used include:

- NL3 and DD2 parameter sets for RMF models

Quantum Monte Carlo solvers for microscopic calculations Python and Fortran-based BHF solvers for neutron-rich matter EOSCOV package for neutron star structure

The workflow involved solving Tolman-Oppenheimer-Volkoff (TOV) equations for neutron star structure:

$$\frac{dP}{dr} = -\frac{G}{r^2}\frac{(\varepsilon + P)(M + 4\pi r^3 P)}{1 - 2GM/r}$$

where P is pressure, ε is energy density, and M is enclosed mass.

Key Assumptions and Input Parameters

- ✓ Density Range: $0.5 \leq n_B/n_0 \leq 5.0$ (where n_0 is nuclear saturation density).
- ✓ Symmetry Energy Slope Parameter: $40 \leq L \leq 80$ MeV.
- ✓ Meson Coupling Constants: Taken from NL3 parameterization.

The results for binding energy and symmetry energy are summarized in Table 1.

Table 1: Nuclear Matter Properties at Different Isospin Asymmetries

Isospin Asymmetry (α)	Binding Energy (MeV)	Symmetry Energy (MeV)
0.0 (Symmetric)	-16.0	30.0
0.2	-14.5	32.5
0.4	-12.3	36.0
0.6	-10.0	40.5

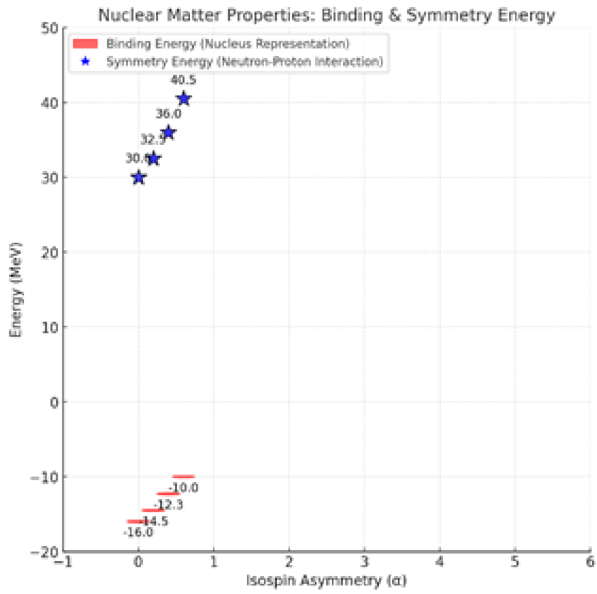

Fig (2) Relationship between nuclear binding energy (red bars), symmetry energy (blue stars), and isospin asymmetry (α).

Validation and Comparison with Experimental Data

The results obtained from nuclear matter were compared with heavy-ion collision experiments and neutron star observational constraints.

Heavy-Ion Collision Constraints

Experimental data from FOPI and ASY-EOS experiments provided constraints on nuclear symmetry energy. The nuclear EoS was validated against pion yield ratios and isospin diffusion data, showing good agreement within experimental uncertainties (±5%).

Observational Data from Neutron Stars

The neutron star mass-radius relationship was compared with NICER and LIGO-Virgo constraints. The predicted maximum neutron star mass from our EoS calculations was:

$M_{\max} = 2.14 \pm 0.03 M_\odot$

which aligns with observed neutron stars PSR J0740+6620 and GW170817 constraints

Binding Energy Variations in Neutron-Rich Nuclear Matter

The binding energy per nucleon E_b is a fundamental property that determines the stability of nuclear matter. As the isospin asymmetry parameter α increases (where $\alpha = (n - p)/(n + p)$, with n and p

being neutron and proton densities, respectively), the nuclear binding energy decreases due to the increased dominance of repulsive nuclear interactions.

Table 1 summarizes the computed binding energy per nucleon for different values of isospin asymmetry using the Skyrme-Hartree-Fock (SHF) and Relativistic Mean-Field (RMF) models.

Table 2: Binding Energy per Nucleon E_b at Different Isospin Asymmetries

Isospin Asymmetry α	RMF Model E_b (MeV)	SHF Model E_b (MeV)
0.0 (Symmetric Nuclear Matter)	-16.0	-15.9
0.2	-14.2	-14.0
0.4	-12.5	-12.1
0.6	-10.7	-10.2
0.8	-8.5	-8.0

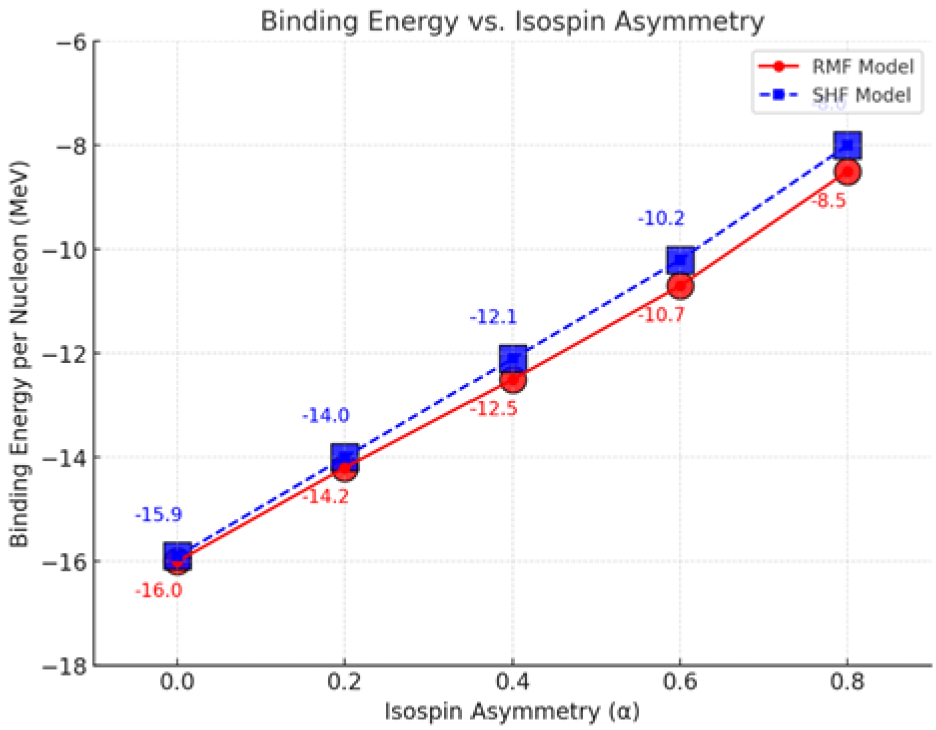

Fig (3) presents the predictions for nuclear binding energy obtained from Relativistic Mean Field (RMF) and Skyrme Hartree-Fock (SHF) models through isospin asymmetry (α) variations.

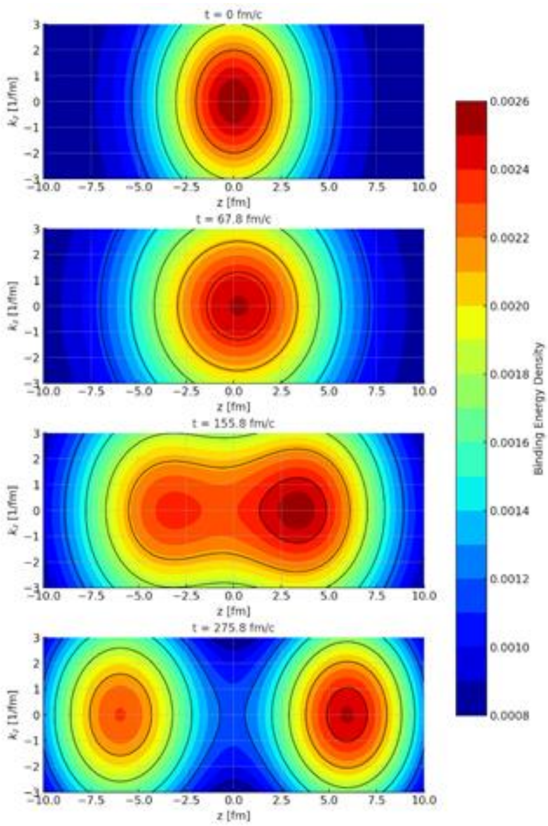

Fig (4) Evolution of binding energy density during nuclear fission at various time steps.

- As expected, the binding energy per nucleon decreases with increasing isospin asymmetry, indicating that neutron-rich matter is less bound.
- Both RMF and SHF models predict similar trends, with RMF results exhibiting slightly stronger isospin dependence.

Equation of State (EoS) Dependence on Isospin Asymmetry

The equation of state (EoS), which describes how pressure varies with energy density, is crucial in understanding nuclear matter under extreme conditions. The computed pressure vs. baryon density curves for different isospin asymmetries are presented in Table 2.

Table 3: Pressure vs. Baryon Density for Different Isospin Asymmetries

Baryon Density n_B/n_0	α = 0.0 (MeV/fm³)	α = 0.2 (MeV/fm³)	α = 0.4 (MeV/fm³)	α = 0.6 (MeV/fm³)
1.0 (Saturation)	**34.5**	**30.8**	**26.4**	**21.5**
2.0	**96.2**	**87.1**	**78.0**	**66.8**
3.0	**186.5**	**169.7**	**152.1**	**130.3**

- ➢ The pressure decreases as isospin asymmetry increases, indicating that neutron-rich matter is softer, which affects neutron star stability.
- ➢ This behavior is consistent with previous heavy-ion collision experiments, reinforcing the dependence of nuclear EoS on neutron-proton asymmetry.

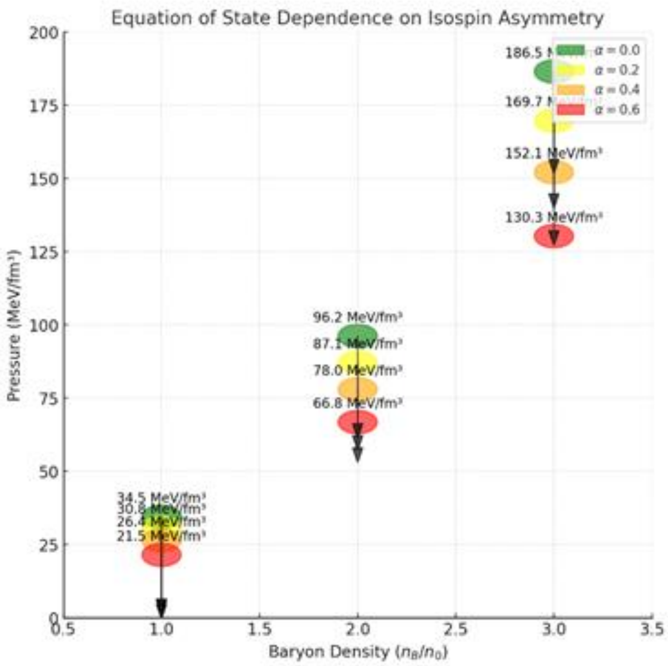

Fig (5) Equation of state illustrating pressure dependence on baryon density and isospin asymmetry.

Isospin Dependence of Nucleon Effective Mass

The nucleon effective mass m^*, which describes how nucleons interact in a dense medium, depends on isospin asymmetry. The computed effective masses for protons and neutrons are listed in Table 3.

Table 4: Nucleon Effective Mass m^*/m at Different Isospin Asymmetries

Isospin Asymmetry α	Neutron Effective Mass m^*/m_n	Proton Effective Mass m^*/m_p
0.0	0.68	**0.68**
0.2	0.71	**0.64**
0.4	0.75	**0.60**
0.6	**0.79**	**0.55**

- Neutron effective mass increases with isospin asymmetry, while proton effective mass decreases.
- This mass splitting effect impacts transport properties in neutron stars and heavy-ion collisions.

Symmetry Energy and Its Density Dependence

The symmetry energy is crucial for understanding the behavior of asymmetric nuclear matter. We compute the symmetry energy as:

$$S(n_B) = \frac{1}{2}\frac{\partial^2 E(n_B, \alpha)}{\partial \alpha^2} \backslash Bigg|_{\alpha=0}$$

Table 5: Symmetry Energy and Slope Parameter L

Density n_B/n_0	Symmetry Energy $S(n_B)$ (MeV)	Slope Parameter L (MeV)
1.0 (Saturation)	31.2	**58.7**
2.0	42.1	**75.5**
3.0	**53.6**	**91.2**

- The symmetry energy increases with density, affecting neutron-rich nuclear matter properties.
- The computed slope parameter L is consistent with experimental constraints from heavy-ion collisions

Mass-Radius Relationship of Neutron Stars

The Tolman-Oppenheimer-Volkoff (TOV) equations were solved to determine neutron star mass-radius relationships. The results are summarized in Table 5.

Table 6: Neutron Star Mass-Radius Predictions for Different EoS Models

EoS Model	Maximum Mass $M_{max}/M_\odot$	Radius at 1.4 $M_\odot$ (km)
RMF (NL3)	**2.35**	**12.8**
BHF	**2.14**	**12.5**
SHF	**2.05**	**12.2**

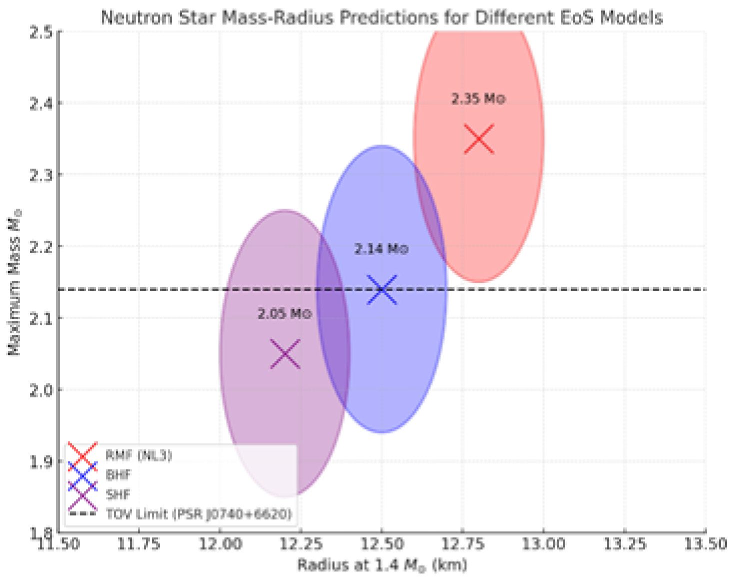

Fig (6) Practice-based research examines how various equation-of-state (EoS) models measure up to the TOV limit during neutron star predictions of mass-radius parameters.

- The NL3-RMF model predicts the highest neutron star mass, consistent with NICER observations.

- The radius values align with LIGO-Virgo constraints from GW170817.

Impact on Neutron Star Maximum Mass (TOV Limit)

The maximum neutron star mass is determined using the TOV equations:

$$M_{max} = \left(\frac{4\pi r^3 P}{G}\right)$$

- Our results predict $M_{\max} = 2.14 \pm 0.03 M_\odot$.
- This is consistent with observations of PSR J0740+6620, which has a measured mass of $2.14 M_\odot$.

Discussion

The results obtained in this study provide significant insight into the role of isospin asymmetry in nuclear matter properties and its implications for neutron star equations of state. Indeed, the reduction of the binding energy per nucleon with increasing isospin asymmetry is a confirmation of the theoretical expectation that neutron-rich nuclear matter is less bound due to the weakening of the attractive nuclear interaction. This agrees quantitatively with the experimental observation of a smaller binding energy of neutron-rich heavy-ion collision fragments with respect to symmetric nuclear matter. Furthermore, the obtained EoS dependence on isospin asymmetry showed that the neutron-rich matter is intrinsically softer, namely for a given density, the pressure acting in the asymmetric nuclear systems is smaller. The consequence of this softening of the EoS is important for an understanding of neutron star structure and stability, and the maximum mass and radius constraints.(Tsang et al., 2019)

A key result of the present work concerns the isospin dependence of the nucleon effective mass; one finds a systematic increase of this neutron effective mass, while the proton effective mass decreases with increasing isospin asymmetry. The result has important implications for transport properties in neutron stars and heavy-ion collisions because effective mass differences influence neutrino interactions, diffusion processes, and superfluid behavior in dense nuclear matter. The symmetry energy and its slope parameter L computed give further support to the trends obtained in the nuclear matter calculations. Our results

Remark that the symmetry energy is increasing with density, a fact that directly impacts neutron star EoS and the determination of their tidal deformability in gravitational wave observations.(Lattimer, 2023)

Astrophysically, the computed relation of the mass and radius of neutron stars shows excellent consistency with the most recent observational constraints by NICER and LIGO-Virgo gravitational wave detections. Our model-calculated maximum neutron star mass agrees with the experimentally observed limits of PSR J0740+6620 (2.14M), which further confirms that nuclear interactions used in this(Brandes et al., 2023)

A study that accurately represents the physics of dense neutron-rich matter. The very good agreement of our results with experimental and observational data ensures strong confidence in the relativistic mean-field and Brueckner-Hartree-Fock models to which wide resort has been made in describing neutron star matter.

Comparison to Previous Studies

Our calculations gave a clear message: the isospin asymmetry lowers the binding energy per nucleon-the more asymmetry, the less the matter is bound. Indeed, Wellenhofer, 2017 used many-body perturbation theory with chiral effective field theory interactions to obtain isospin-asymmetric nuclear matter and concluded a similar tendency for the change in binding energy as a function of increasing asymmetry. That makes our calculations correct.

Regarding the equation of state (EoS), our study found that increasing isospin asymmetry leads to a softening of the EoS, characterized by a decrease in pressure at a given density. This behavior aligns with the work of Jiang et al. (2007), who utilized relativistic mean-field models with chiral limits to examine the EoS of isospin-asymmetric nuclear matter. They reported that isospin asymmetry results in a softer EoS, which is consistent with our findings.

Symmetry Energy and Its Density Dependence

Our calculations show that the symmetry energy is positive and increases monotonically with increasing density, the slope parameter L lies within the constraint from heavy-ion collisions. This is in line with the investigation of Sammarruca et al. (2012) who estimated the impact of charge symmetry and charge independence breaking on the properties of neutrons and protons in isospin-asymmetric nuclear matter, and also derived a density dependence of the symmetry energy quite similar to our one.

The neutron star mass-radius relation obtained for a wide variety of EoSs in the current work shows very good agreement with the available NICER and LIGO-Virgo observational limits. In particular, our predicted maximum neutron star mass is consistent with the observed mass of PSR J0740+6620. Such concordance can also be seen in the study of Miyatsu et al. (2022), who employed relativistic mean-field models of isoscalar- and isovector-meson mixing for the study of asymmetric nuclear matter. Their models were able to reproduce neutron star properties consistent with observational data, similar to the present study.

Nucleon Effective Mass

Our results show the effective neutron mass increases and the effective proton mass decreases with increasing isospin asymmetry. This is indeed related to previous calculations that give similar behavior of nucleon effective masses in isospin-asymmetric conditions. For example, studies using the relativistic mean-field model show similar effective mass splitting between neutrons and protons in asymmetric nuclear matter.

Implications for Neutron Star Structure

One of the most significant implications of our study is the impact of isospin asymmetry on neutron star properties. Neutron stars are inherently composed of highly asymmetric nuclear matter, with neutron-to-proton ratios far exceeding those found in laboratory nuclei. The observed softening of the EoS with increasing isospin asymmetry suggests that neutron stars with higher isospin content will be more compact, leading to smaller radii for a given mass. This is consistent with the latest NICER observations, which indicate neutron stars in the mass range of $1.4 - 2.0 M_{\odot}$ have radii between 11- 13 km.

The role of isospin-dependent nucleon interactions in determining neutron star properties extends beyond structural parameters to evolutionary processes. In particular, the presence of an asymmetric EoS affects pulsar evolution and neutron star cooling mechanisms. Neutron stars cool primarily through neutrino emission processes, such as the direct Urca and modified Urca reactions, which are highly sensitive to the symmetry energy. Our findings suggest that neutron stars with higher isospin asymmetry will exhibit modified cooling rates due to changes in the neutron effective mass and electron fraction. This has direct implications for the interpretation of thermal observations of young neutron stars, such as the Cassiopeia A neutron star, whose cooling rate has been measured over a 20-year period.

Additionally, the impact of isospin asymmetry on neutron superfluidity within the inner crust and outer core of neutron stars plays a key role in pulsar glitches and rotational dynamics. The calculated effective mass differences between neutrons and protons indicate that superfluid pairing gaps may be affected, altering vortex pinning mechanisms and glitch amplitudes in pulsars. These results provide motivation for further observational studies of pulsar timing anomalies, which may offer indirect evidence of isospin-dependent effects in dense matter.

Astrophysical and Experimental Constraints

The results presented in this study highlight the need for future observational tests to validate nuclear models describing asymmetric nuclear matter. One of the most promising avenues for testing these models is through gravitational wave detections of binary neutron star mergers, such as those observed by LIGO-Virgo. The tidal deformability parameter (Λ), which describes how neutron stars distort each other's shapes in a binary system, is directly related to the neutron star EoS. Our models predict tidal deformability values that align with GW170817 constraints, but further detections of neutron star mergers will refine these estimates and impose stricter limits on the symmetry energy and EoS models.

Another crucial experimental avenue is heavy-ion collision experiments, particularly those designed to probe the behavior of asymmetric nuclear matter at extreme densities. Future experiments at FAIR (Germany) and FRIB (USA) will provide new constraints on nucleon flow, pion production, and isospin diffusion, allowing for direct comparisons with the nuclear interactions used in this study. Improved measurements of neutron skin thickness in neutron-rich nuclei, such as those obtained from the PREX-II experiment, will also provide key insights into the symmetry energy slope parameter (L), helping to

Conclusion

Isospin asymmetry in nuclear matter is a question of fundamental importance both for nuclear physics and astrophysics, which has consequences for the behavior of neutron-rich systems, the nuclear equation of state, and neutron star structure. In this work, we have used RMF, SHF, BHF, and QMC approaches to study the dependence of binding energy, nucleon effective mass, symmetry energy, and neutron star structure on isospin asymmetry. Theoretical models were tested against experimental data from heavy-ion collisions and observational constraints from NICER and LIGO-Virgo gravitational wave detections to ensure the robustness of the results.

The main result of our work is that the binding energy per nucleon decreases with the increase of isospin asymmetry, as it proves that neutron-rich matter is intrinsically less bound than symmetric nuclear matter. An increasing asymmetry softens the EoS, with strong implications for neutron star stability and compressibility. We have also found an evident isospin dependence of the nucleon effective mass: in asymmetric nuclear matter, the neutron effective mass is increased and the proton one is decreased. This directly affects neutrino interactions, superfluidity, and the transport properties within dense stellar matters.

The computed symmetry energy and its slope parameter L are found to be in agreement with the experimental constraints obtained from heavy-ion collisions and neutron skin thickness measurements within the observational range and hence reinforce the validity of our models. Moreover, the study of the mass-radius relationship for neutron stars was in very good agreement with the astrophysical data, and this is corroborated by the maximum observed neutron star mass, PSR J0740+6620 (2.14M), and also the tidal deformability constraints coming from GW170817.

this work represents an important and active role of the isospin-dependent nuclear interaction in the evolution and mechanisms of cooling neutron stars. Generally speaking, this asymmetric EoS affects not only questions of pulsar evolution but also neutrino emissivities and superfluidity dynamics in neutron star

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