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# Some Results Generalized Curvature Tensor of Viesman-Grey Manifold

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**Abstract:** General Background: The study of differential geometry plays a crucial role in understanding the structural properties of various manifolds, particularly in the context of Riemannian and Hermitian geometry. Specific Background: The Viesman-Grey manifold is a significant class in this field, characterized by its unique conharmonic curvature tensor. Previous studies have examined nearly Hermitian manifolds, conformal invariants, and curvature identities, yet a comprehensive analysis of the generalized conharmonic curvature tensor in Viesman-Grey manifolds remains insufficient. Knowledge Gap: Despite extensive research on curvature tensors in different geometric structures, the specific characteristics and classification of the conharmonic curvature tensor in generalized Viesman-Grey manifolds require further exploration. Aims: This study aims to investigate the properties of the generalized conharmonic curvature tensor in Viesman-Grey manifolds, examining its structure and defining new classifications. Results: The research identifies various classes of Viesman-Grey manifolds based on the components of the conharmonic curvature tensor and establishes their interrelations. The study provides equations characterizing different classes and proves specific properties of the generalized tensor. Novelty: This study introduces new classifications of generalized Viesman-Grey manifolds, offering an extended perspective on conharmonic curvature tensor properties and their implications in differential geometry. Implications: The findings contribute to the theoretical development of Riemannian geometry by refining the classification of nearly Hermitian and Viesman-Grey manifolds. These insights may support future studies in geometric structures, tensor analysis, and mathematical physics.

**Keywords:** Generalized conharmonic tensor, Viesman-Grey manifold, curvature tensor, differential geometry, nearly Hermitian manifold, Riemannian space.

## 1. Introduction

As it demonstrates how differential geometry is constructed, the nearly Hermitian manifold (AH-manifold) is regarded as one of the most important components of differential geometry. Banar'u [1] efficiently reclassified 16 classes of nearly manifolds in 1993 utilizing structure & virtual tensors. Kirechenko's , one of sixteen locally conformal kahler manifold classes, will be studied in this paper. Riemannian space studies of the surface of the previously described conharmonic tensor.Three different kinds of nearly Hermitian manifolds were distinguished by Gray, and each of them has a Riemannian curvture expression tensor. These classes are identified by $R_1$, $R_2$, and $R_3$. The class $R_3$ corresponds to the RK-manifold. The key to comprehending differential geometrical features, according to $R_1$, $R_2$, and $R_3$, is the identities of Kahler manifolds that satisfy them. Gray [2]. The the Riemann tensor is composed of the following elements:

Ŕ1:<( $(x_1, x_2, x_3, x_4)$ >= Ŕ1:<( JX1, JX2,) $x_3, x_4$>

Ŕ2:<( $(x_1, x_2)x_3, x_4$>= < Ŕ(JX1, JX2) $x_3, x_4$>+ < Ŕ(JX1, $x_2$) JX3, $x_4$>+ < Ŕ(JX1, $x_2, x_3$, JX4>;

Ŕ3:<( $(x_1, x_2)x_3, x_4$>= < Ŕ($(x_1, x_2)x_3, x_4$>+ < Ŕ(JX1, JX2) JX3, JX4>;

AH-structures have a tensor R that meets the identity Ŕi if the name is any class of H-structures, θ ∩ $Ŕ_i$=θ where i = 1,2, 3, [9] . C ⊂ $Ŕ_1$ ⊂ $Ŕ_2$ ⊂ $Ŕ_3$ is an established fact. It is reasonable to assume that the $Ŕ_1$and $Ŕ_3$ manifold classes are the closest to the Kahler manifold class among AH-manifolds with different $Ŕ_i$ geometrical and topological features. In 1996, Kirichenko and Eshova conducted research on the conformational invariant of the GV-manifold of class. Rakees [3] examined the local conformal kahler of class $Ŕ_3$ in 2008, provided that the Lie form's element is valid.

This is able to be expressed the complex concircular tensor of particular classes of nearly hermitian manifolds was studied by Saadan in 2011. Jawad studied the nearly Kehler manifold of the class Parakehler $R_1$. In 2018, Dhabia'a was studiedthe Generalized Conharmonic manifold Class Nearly Kahler. In the article studying the type of generalized conharmonic of the Viesman-Grey manifold[4].

**Materials and Methods**

**2. Preliminaries**

The AH-structure and the tensor R are examples of class structures that meet the identity $Ŕ_i$ , i=1, 2, 3, Then comes the designation for an AH-structure subclass. It is commonly known that C ⊂ $Ŕ_1$ ⊂ $Ŕ_2$ ⊂ $Ŕ_3$ .Therefore, it is reasonable to assume that the GV-manifold class will be closest to the equivalent geometric and topology AH-manifold. $Ŕ_1$ , $R_2$ and $Ŕ_3$. In the study of the GV-manifold, artists such as Grey and Venhacke [5], Waston, and Venhecke were also considered, as was the manifold of class $Ŕ_2$. Let the 2n-dimensional GV-manifold be (M,J,g).

**Definition 2.1 [4]**

A structure {J, g=<.,.> } is a Veisman-Gray structure in the following manner:

$\nabla_X$ (F)( X ,Y)=(-1)/(2(Ƞ-1)){< X, X, >SF(Y)-< X , Y >δF(X)- <JX,Y >SF(JX)}……(1).

Where ∇ is the Riemannian connector by g, F(X,Y)=< JX, Y > is the Kohler form, ʃ is a derivative, and X, Y, and Z belong to X(M). In the context of a killing farm,

an AH-structure (J, g = <... >) is referred to as $W_1$ or nearly Kehler (NK-structure) $\nabla_X$(J)=0 ;X ∊X(M)……(2).

Structure of AH (J, g=<.;.>) a locally conforming Kehler structure (LCK-structure), or brief $W_4$ class structure, if $\nabla_X$(F)(Y, Z)=(-1)/2(Ƞ-1) {<X,Y>SF(Z)-<X, Z>SF(Y)- <X,JY >SF(JZ)+X ,JZ>SF(JY)}.

**Properties2.2[1]**

Assume that the Viesman-Grey Manifold is (M; J, g). Additionally, it possesses every quality of a tensor of curvature:

V(Ẋ, Ẏ, Ẓ , Ẇ)= -V(Ẏ, Ẋ, Ẓ , Ẇ);

1) V(Ẋ, Ẏ, Ẓ, Ẇ)= -V(Ẏ, Ẋ, Ẇ, Ẓ);
2) V(Ẋ, Ẏ, Ẓ, Ẇ) + V(Ẏ, Ẓ, X, Ẇ) + V(Ẓ, Ẋ , Ẏ, Ẇ) = 0;
3) V(Ẋ, Ẏ, Ẓ, Ẇ)= -V(Ẓ, Ẇ, Ẋ, Ẏ);

$\forall\ \dot{X}, \dot{Y}, Z, \dot{W} \in \dot{X}(M)$;

**Definition 2.3**

$(GV)i$, $\forall\ i = 0, 1, \ldots, 7$ is the GT-manifold for which $(GV)i=0$ is the GV-manifold of classes.

**Theorem 2.4**

Neutral equations for GV-manifolds are described by this theorem. Compounds of the class conharmonic curvature:

1) GV- manifolds of class $(GV)_0$ characterized:

$GV(X_1, X_2) X_3- GV(X_1, JX_2) JX_3- GV (JX_1, X_2) JX_3- GV(JX_1, JX_2) X_3-JGV(X_1, X_2) JX_3-JGV(JX_1, X_2) X_3- JGV(JX_1, X_2) JX_3+JGV(JX_1, JX_2) JX_3; X_1,X_2,X_3 \in X(M)$….(3)

2) GT- manifold of class $(GT)_1$ characterized by :

$GV(X_1, X_2) X_3+ GV(X_1, JX_2)JX_3- GV(JX_1, x_2) JX_3+ GV(JX_1, JX_2) X_3+JGV(X_1, X_2) JX_3-JGV(JX_1, X_2) X_3- JGV(JX_1, X_2) JX_3+JGV(JX_1, JX_2) JX_3; X_1,X_2,X_3 \in X(M)$….(4)

3) GV-manifold of class $(GV)_2$ characterized by :

$GV(X_1,X_2)X_3 -GV(X_1,JX_2) JX_3+GV(JX_1,X_2) JX_3+ GV(JX_1, JX_2) X_3-JGV(X_1, X_2) JX_3-JGV(JX_1, X_2) X_3+ JGV(JX_1, X_2) JX_3-JGV(JX_1, JX_2) JX_3$;….(5)

4) GV-manifold of class $(GV)_3$ characterized by :

$GV(X_1,X_2)X_3+GV(X_1,JX_2)JX_3+GV(JX_1,X_2)JX_3-GV(JX_1,JX_2)X_3-JGV(X_1,X_2) JX_3+JGV(JX_1, X_2) X_3+JGV(JX_1, X_2) JX_3+JGV(JX_1, JX_2) JX_3; X_1,X_2,X_3 \in X(M)$….(6)

5) GV-manifold of class$(GV)_4$ characterized by :

$GV(X_1,X_2)X_3+GV(X_1,JX_2)JX_3+GV(JX_1,X_2)JX_3-GV(JX_1,JX_2)X_3+JGV(X_1,X_2)JX_3-JGV(JX_1, X_2) X_3- JGV(JX_1, X_2) JX_3-JGV(JX_1, JX_2) JX_3 ; X_1,X_2,X_3 \in X(M)$….(7)

6) GT-manifold of class$(GV)_5$ characterized by :

$GT(X_1,X_2)X_3-GV(X_1,JX_2)JX_3+GV(JX_1,X_2)JX_3+GV(JX_1,JX_2)X_3+JGV(X_1,X_2) JX_3+JGV(JX_1, X_2) X_3- JGV(JX_1, X_2) JX_3+JGV(JX_1, JX_2) JX_3 ; X_1,X_2,X_3 \in X(M)$….(8)

7) GV-manifold of class$(GT)_6$ characterized by :

$GV(X_1, X_2) X_3+ GV(X_1, JX_2) JX_3-GV(JX_1, X_2) JX_3+ GV(JX_1, JX_2) X_3+JGV(X_1, X_2) JX_3-JGV(JX_1, X_2) X_3+JGV (JX_1, X_2) JX_3+JGV(JX_1, JX_2) JX_3 ; X_1,X_2,X_3 \in X(M)$….(9)

8) GV-manifold of class$(GV)_7$ characterized by :

$GV(X_1,X_2) X_3-GV(X_1, JX_2) JX_3- GV(JX_1, X_2) JX_3- GV(JX_1, JX_2) X_3+JGV(X_1, X_2) JX_3+JGV(JX_1, X_2) X_3+ JGV(JX_1, X_2) JX_3-JGV(JX_1, JX_2) JX_3 ; X_1,X_2,X_3 \in X(M)$….(10)

**Theorem 2.5[1]**

The following describes elements of the GV-manifolds' generalization conharmonic curvature in the nearby V-structure:

$GV_{ijkl}=R_{ijkl}-1/2(n-1) [g_{ik}\, r(GV)_{jl}+g_{jl} r(GV)_{ik}-g_{il}\, r(GV)_{jk}-g_{jk}\, r(GV)_{il}]$.

Then

1) $GV_{a\hat{b}\hat{c}d}=0-1/2(n-1)\ \{(S^c{}_a)(-A^{bk}{}_{dk}+ B^{bh}{}_d B^k{}_{hk} +1/2\ \alpha^{[b}{}_{[d}\ S^{\ k]}{}_{k]})+(S^c{}_a))(-A^{ak}{}_{ck}+ B^{ah}{}_c B_{hk}{}^k+1/2\ \alpha^{[a}{}_{[c}\ S^{k]}{}_{k]})- (S^a{}_d) (-A^{bk}{}_{ck}+ B^{bh}{}_c B_{hk}{}^k +1/2\ \alpha^{[b}{}_{[c}\ S^{\ k]}{}_{k]})- (S^a{}_d) (-A^{ak}{}_{dk}+ B^{ah}{}_d B_{hk}{}^k +1/2\ \alpha^{[a}{}_{[d}\ S^{\ k]}{}_{k]})\}$

2) $GV_{\hat{a}bc\hat{d}}= (-A^{ac}{}_{bd}+ B^{ah}{}_b B_{hd}{}^c +1/2\ \alpha^{[a}{}_{[b}\ S^{\ c]}{}_{d]})+ 1/2(n-1)\ (S^a{}_d) (-A^{ck}{}_{bk}+ B^{ch}{}_b B_{hk}{}^k +1/2\ \alpha^{[c}{}_{[b}\ S^{\ k]}{}_{k]})- (S^c{}_b) (-A^{ak}{}_{dk}+ B^{ah}{}_d B_{hk}{}^k +1/2\ \alpha^{[a}{}_{[d}\ S^{\ k]}{}_{k]})\}$

3) $GV_{\hat{a}bc\hat{d}} = (-A^{ad}{}_{bc}+ B^{ah}{}_b B_{hc}{}^d +1/2\ \alpha^{[a}{}_{[b}\ S^{\ d]}{}_{c]})- 1/2(n-1)\ (S^a{}_c) (-A^{dk}{}_{bk}+ B^{dh}{}_b B_{hk}{}^k +1/2\ \alpha^{[d}{}_{[b}\ S^{\ k]}{}_{k]})+( S^c{}_b) (-A^{ak}{}_{ck}+ B^{ah}{}_c B_{hk}{}^k +1/2\ \alpha^{[a}{}_{[c}\ S^{k]}{}_{k]})\}$

**Theorem 2.6**

Our inclusive relations are as follows:

i) $(GV)_0 = (GV)_3$

ii) $(GV)_1 = (GV)_2$

iii) $(GV)_4 = (GV)_7$

iv) $(GV)_5 = (GV)_6$

**Proof :**we shall prove (i)

$(GV)_0=GT(X_1,X_2)X_3-GV(X_1,JX_2)JX_3-GV(JX_1,X_2)JX_3-GV(JX_1,JX_2)X_3-JGV(X_1,X_2)JX_3-JGV(JX_1, X_2)\ X_3-\ JGV(JX_1,\ X_2)\ JX_3+JGV(JX_1,\ JX_2)\ JX_3$ ….(11)

$(GT)_0=GV(X_1,X_2)X_3-GV(x_1,-\sqrt{(-1)}JX_2)(-\sqrt{(-1)})JX_3-GV(-\sqrt{(-1)}JX_1,X_2)(-\sqrt{(-1)})JX_3-GV(-\sqrt{(1)}JX_1,-\sqrt{(-1)}JX_2)x_3-(-\sqrt{(-1)})JGV(x_1,x_2)(-\sqrt{(-1)})JX_3-(-\sqrt{(-1)})JGV(-\sqrt{(-1)}JX_1,x_2)x_3-(-\sqrt{(-1)})JGV(-\sqrt{(-1)}\ JX_1,X_2)(-\sqrt{(-1)})\ JX_3+(-\sqrt{(-1)})JGV(-\sqrt{(-1)}\ JX_1,\ -\sqrt{(-1)}\ JX_2)\ (-\sqrt{(-1)})\ JX_3$

$(GV)_0=GV(X_1,X_2)X_3-GV(X_1,JX_2)JX_3-GV(JX_1,X_2)JX_3+GV(JX_1,JX_2)X_3-JGV(X_1,X_2)JX_3-JGV(JX_1, X_2)\ X_3-\ JGV(JX_1,\ X_2)\ JX_3+JGV(JX_1,\ JX_2)\ JX_3$ ….(12)

We obtain from (11) and (12):

$(GV)_0=GV(X_1,X_2)X_3-GV(JX_1,JX_2)X_3-JGV(X_1,X_2)\ JX_3+JGV(JX_1,\ JX_2)\ JX_3$ ….(13)

$(GV)_3=GT(X_1,X_2)X_3+GV(X_1,JX_2)JX_3+GV(JX_1,X_2)JX_3-GV(JX_1,JX_2)X_3-JGV(X_1,X_2)\ JX_3+JGV(JX_1, X_2)\ X_3+JGV(JX_1,\ X_2)\ JX_3+JGV(JX_1,\ JX_2)\ JX_3$ ….(14)

$(GV)_3=GV(X_1,X_2)X_3+GV(X_1,-\sqrt{(-1)}JX_2)(-\sqrt{(-1)})JX_3+GV(-\sqrt{(-1)}JX_1,X_2)(-\sqrt{(-1)})\ JX_3-GV(-\sqrt{(-1)}JX_1,-\sqrt{(-1)}JX_2)X_3-(-\sqrt{(-1)})JGV(X_1,X_2)(-\sqrt{(-1)})JX_3+(-\sqrt{(-1)})JGV(-1)JX_1,X_2)X_3+(-\sqrt{(-1)})JGV(-\sqrt{(-1)}\ JX_1,X_2)(-\sqrt{(-1)})\ JX_3+(-\sqrt{(-1)})JGV(-\sqrt{(-1)}\ JX_1,\ -\sqrt{(-1)}\ JX_2)\ (-\sqrt{(-1)})\ JX_3$

$(GV)_3=GV(X_1,X_2)X_3-GV(X_1,JX_2)JX_3-GV(JX_1,X_2)JX_3-GV(JX_1,JX_2)X_3-JGV(X_1,X_2)JX_3-JGV(JX_1, X_2)\ X_3-\ JGV(JX_1,\ X_2)\ JX_3+JGV(JX_1,\ JX_2)\ JX_3$ ….(15)

We obtain from (14) and (15):

$(GV)_3=GV(X_1,X_2)X_3-GV(JX_1,\ JX_2)X_3-JGV(X_1,X_2)\ JX_3+JGV(JX_1,\ JX_2)\ JX_3$ ….(16)

We obtain from (13) and (16) we get $(GV)_0=(GV)_3$:

In the same way we prove:

$(GV)_1 = (GV)_2$

$(GV)_4 = (GV)_7$

$(GV)_5 = (GV)_6$

**Definition 2.7**

The letters (M, J, g) stand for a class manifold in such a way that:

1) $(GV)_1$ if $< GV(X_1,X_2)X_3,X_4> = GV(X_1,X_2)\ JX_3,\ JX_4>$;

2) $(GT)_2$ if $< GV(x_1;x_2)x_3,x_4> = < GV(JX_1,\ JX_2)X_3;X_4> + < GV(JX_1,X_2)\ JX_3,\ JX_4> + < GV(JX_1,\ X_2)\ X_3,\ JX_4>$;

3) $(GV)_3$ if $< GV(X_1,X_2)X_3,X_4 = < GV(JX_1,\ JX_2)\ JX_3,\ JX_4>$;

**Theorem 2.8**

1) $\theta$ - denote a class's structure of $(GV)_3$;

2) $(GV)_0 = 0$ and

(3) In adjoined G-structure space, the values of $GV^{a}_{bcd} = = 0$ make sense.

**Proof :**

If we consider the θ-structure of class $(GV)_3$, it is unquestionably equivalent to identity.

$(GV)(X_1,X_2)X_3)+J(GV)(JX_1, JX_2) JX_3)=0$;

By of a continuum tensor:

$GV(X_l,X_2)X_3=(GV)_0(X_l,X_2)X_3-(GV)_1(X_l,X_2)X_3-(GV)_2(X_l, X_2)X_3-(GV)_3(X_1,X_2) X_3-(GV)_4 (X_l,X_2)X_3-(GV)_5(X_l,X_2)X_3-(GV)_6(X_1,X_2) X_3+(GV)_7(X_l,X_2)X_3$

$J_oGV(X_1,X_2)X_3=J_o(GV)_0(JX_1,JX_2)JX_3+J_o(GV)_1(JX_1,JX_2)JX_3+J_o(GV)_2(JX_1,JX_2) JX_3+J_o(GV)_3(JX_1,JX_2)JX_3+J_o(GV)_4(JX_1,JX_2)JX_3+J_o(GV)_5(JX_1,JX_2)JX_3+J_o(GV)_6(JX_1,JX_2)_3+J_o(GV)_7(JX_1,JX_2)JX_3=(GV)_0(X_1,X_2)X_3-(GV)_1(X_1,X_2)X_3-(GV)_2(X_1,X_2)X_3-(GV)_3(X_l,X_2)X_3-(GV)_4(X_l,X_2)X_3+(GV)_5(X_l,X_2)X_3+(GV)_6(X_l,X_2)X_3-(GV)_7(X_l,X_2)X_3$

These identifiers will be defined using the following:

$(GV)(X_1,X_2)X_3+J(GV)(JX_1,JX_2)JX_3+(GV)_0(X_1,X_2) X_3+(GV)_3(X_1,X_2) X_3+(GV)_5(X_1,X_2)_3+(GV)_6(X_1,X_2) X_3$

With resources, an identity,$(GV)(X_1,X_2)X_3+J(GV)( JX_1, JX_2) JX_3=0$ is comparable with it $(GV)_0(X_1,X_2) X_3+(GV)_3(X_1,X_2) X_3+(GV)_5(X_1,X_2) X_3+(GV)_6(X_1,X_2) X_3$

This single identity can represent multiple identities.

$(GV)_0=(GV)_3=(GV)_5=(GV)_6=0$

Based on characteristics (2.2), Furthermore, the given identity of adjoined V-structuer are equivalent as relationships.

$$\mathrm{GV}^{\mathrm{a}}_{\mathrm{bcd}} = GV^{a}_{b\hat{c}\hat{d}} = GV^{a}_{\hat{b}\hat{c}\hat{d}} = 0$$

i.e. identity $(GV)(X_1,X_2)X_3=0$.

**Theorem2.9**

Let the V-structure be represented by = (J, g =.,. >). They are interchangeable.

(1) θ -structure of a classes$(GV)_2$;

(2) $(GV)_0=(GV)_7=0$;

(3) Are the following reasonable on the associated G-structure identification space: $\mathrm{GV}^{\mathrm{a}}_{\mathrm{bcd}}=\mathrm{GV}^{\mathrm{a}}_{\mathrm{b}\hat{\mathrm{c}}\hat{\mathrm{d}}}=0$

**Theorem 2.10**

Let the GT-structure be represented by = (J, g=.,. >). The same claims:

(1) θ -structure of classes$(GV)_1$;

(2) $(GV)_0=(GV)_4=(GV)_7=0$; and

(3) Are the following reasonable on the associated G-structure identification space: $\mathrm{GV}^{\mathrm{a}}_{\mathrm{bcd}}=\mathrm{GV}^{\mathrm{a}}_{\mathrm{b}\hat{\mathrm{c}}\hat{\mathrm{d}}}=0$

**Proof :**

Let a class's θ-structure $(GT)_3$. Clearly, it is synonymous with identity.

$<GV(X_1,X_2) X_3,X_4>=<GV(X_1,X_2) JX_3, JX_4>$, and we obtain:

$<GV(X_1,X_2)X_3,X_4>=<JGV(X_1,X_2)X_3, JX_4>=0$;

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By of a continuum tensor:

1 $(GV)_3(X_1,X_2)X_3=(GV)_0(X_1,X_2)X_3+(GV)_1(X_1,X_2)\,X_3+(GT)_2(X_1,X_2)\,X_3+(GV)_3(X_1,X_2)\,X_3+$ $(GV)_4(X_1,X_2)X_3+(GV)_5(X_1,X_2)\,X_3+(GV)_6(X_1,X_2)\,X_3+(GV)_7(X_1,X_2)X_3$

2 $J_o(GV)(X,X_2)X_3=J_o(GV)_0(JX_1,JX_2)\,JX_3+J_o(GV)_1(JX_1,JX_2)JX_3+J_o(GV)_2(JX_1,JX_2)JX_3$

3 $+J_o(GV)_3(JX_1,JX_2)JX_3+J_o(GV)_4(JX_1,JX_2)JX_3+J_o(GV)_5(JX_1,JX_2)JX_3+J_o(GV)_6(JX_1,{}_3)_7(JX_1,JX_2)JX_3$ $=(GV)_0(X_1,X_2)X_3-(GV)_1(X_1,X_2)\,X_3-(GV)_2(X_1,X_2)\,X_3$ $(GV)_3(X_1,X_2)\,X_3+(GV)_4(X_1,X_2)\,X_3-(GV)_5(X_1,X_2)\,X_3-$ $(GV)_6(X_1,X_2)\,X_3+(GV)_7(X_1,X_2)X_3$

Placing (1) and (2) in

$(GV)(X_1,X_2)X_3+J(GV)_3(X_1,X_2)\,JX_3$, implies that this property is the same as that

$(GV)_0(X_1,X_2)\,X_3+(GV)_4+(X_1,X_2)\,X_3+(GV)_7(X_1,X_2)\,X_3=0$

Additionally, this identity is interchangeable for other names. $(GV)_0+(GV)_4+(GV)_7=0$

Accordingly, the given identity in space of the adjoined V-structure are the same as

$$GV^{a}_{bcd} = GV^{a}_{b\hat{c}d} = GV^{a}_{\hat{b}\hat{c}\hat{d}} = 0.$$

**Theorem 2.11**

Assume θ = (J, g = <. , . >) is a V-structure, then the following class inclusion.

$(GV)_1 \subset (GV)_2 \subset (GV)_3$ are rational.

**Proof :**

Let's $(GV)_1$ is a kind of θ-structure. It is equivalent to $(GV)_0 = (GV)_4 = (GV) = 0$ according to theory (2.10). Classes $(GV)_0 = (GV)_7 = 0$is the same as class $(GV)_2$ due to theorem (2.9). Then $(GV)_1 \subset (GV)_2$.Also, class $(GT)_3$ is equivalent to class $(GV)_0$. As indicated in theorem (2.7), we obtain: $(GV)_1 \subset (GV)_2 \subset (GV)_3$.

**Result and Discussion**

The study of generalized conharmonic curvature tensor in Viesman-Grey manifolds provides a deeper understanding of the geometric structures within the framework of differential geometry. The research presents a classification of these manifolds based on their curvature tensor properties, offering new insights into the mathematical foundation of these structures. The results emphasize the interdependence between different classes of Viesman-Grey manifolds, providing a more refined classification system that builds upon previous studies in the field. This classification framework enhances the theoretical knowledge of Riemannian geometry and contributes to a more profound understanding of nearly Hermitian manifolds and their variations.

One of the significant findings of this study is the formulation of key equations that define the relationship between the components of generalized conharmonic tensors. These equations provide a systematic way to analyze the structural characteristics of Viesman-Grey manifolds and their geometric behaviors. The study demonstrates that the conharmonic curvature tensor plays a fundamental role in describing the intrinsic and extrinsic properties of these manifolds. Moreover, by refining these mathematical formulations, the research offers a robust framework that can be used for further exploration in tensor analysis and differential geometric modeling.

Theoretical and Practical Implications

The theoretical contributions of this study lie in its advancement of geometric tensor analysis, particularly in the context of generalized conharmonic tensors. The research extends previous work on nearly Hermitian manifolds and introduces a more detailed examination of curvature tensors within the Viesman-Grey manifold classification. The results provide valuable insights for researchers working in mathematical physics, geometric topology, and tensor calculus, as they establish a refined basis for analyzing curvature tensors in different mathematical contexts.

From a practical perspective, the findings have implications in applied mathematics and theoretical physics, particularly in cosmology, relativity, and geometric mechanics. The refinement of curvature tensor classifications may contribute to the study of spacetime structures, allowing for a better understanding of how curvature affects gravitational dynamics in higher-dimensional models. Moreover, applications in computational geometry and machine learning can benefit from the enhanced understanding of tensor relationships, particularly in areas such as image recognition, shape analysis, and topological data processing.

Addressing the Knowledge Gap

Despite the extensive research on conharmonic curvature tensors and their applications, significant knowledge gaps remain in the classification and practical application of Viesman-Grey manifolds. This study addresses some of these gaps by providing a more detailed and systematic classification of generalized conharmonic tensors. However, further research is needed to explore the physical applications of these tensors in advanced geometric and mathematical physics models.

A crucial gap in the literature is the lack of higher-dimensional generalizations of the findings presented in this study. While the research provides a strong mathematical foundation, the applicability of Viesman-Grey manifolds in higher-dimensional spaces remains underexplored. Additionally, the interaction between conharmonic tensors and other curvature tensors in generalized Riemannian and Kahler manifolds requires further investigation to fully understand their mathematical significance.

Further Research Directions

Given the theoretical and practical importance of the findings, future research should focus on several key areas:

Higher-Dimensional Generalizations – Extending the classifications of generalized conharmonic curvature tensors to higher-dimensional Viesman-Grey manifolds and analyzing their geometric properties in more complex spaces.

Application in Mathematical Physics – Investigating the role of Viesman-Grey manifolds in general relativity and quantum field theory, particularly in the context of spacetime curvature and black hole dynamics.

Computational and Algorithmic Approaches – Developing numerical and computational models based on the proposed classifications to facilitate the application of tensor analysis in data science, artificial intelligence, and computational geometry.

Interdisciplinary Applications – Exploring the relevance of curvature tensors in other disciplines, such as biomechanics, structural engineering, and complex systems modeling, where geometric and topological properties play a critical role.

Comparative Studies – Conducting comparative analyses between Viesman-Grey manifolds and other generalized manifolds to understand the uniqueness of their curvature properties and potential interdisciplinary applications.

## Conclusion

This research contributes to the ongoing exploration of differential geometry by introducing a systematic classification of generalized conharmonic curvature tensors in Viesman-Grey manifolds. The results provide a strong mathematical foundation that enhances the understanding of these structures while offering potential applications in physics, engineering, and computational modeling. Future studies should focus on higher-dimensional generalizations, interdisciplinary applications, and numerical implementations to further extend the impact of curvature tensor analysis in both theoretical and applied contexts.

## Conclusion

This study provides a comprehensive analysis of the generalized conharmonic curvature tensor within Viesman-Grey manifolds, offering new classifications and mathematical formulations that enhance the understanding of their structural properties. The findings highlight the interrelations between different classes of Viesman-Grey manifolds, establishing critical equations that characterize their geometric nature. Additionally, the study demonstrates how conharmonic curvature tensor properties influence the broader framework of differential geometry, particularly in relation to nearly Hermitian manifolds. These results have significant implications for advancing the theoretical foundation of Riemannian geometry, contributing to the refinement of curvature tensor analysis in geometric structures. The insights gained from this study may further aid in mathematical physics applications and computational modeling of manifold-based systems. Future research should focus on extending these classifications to higher-dimensional manifolds, exploring their implications in complex differential geometry, and investigating potential applications in mathematical physics and theoretical cosmology to deepen the understanding of curvature dynamics in advanced geometric spaces.

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