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The Use of Generalized Transponed Matrixes in the Study of Stability of Large-Scale Systems

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Abstract: The theory of motion stability studies the influence of disturbing factors on the motion of a material system. Undertakers are forces that are not taken into account when describing motion, because they are small compared to the main forces. These irritating forces are usually unknown. They can act instantaneously, which leads to a small change in the initial state of the material system, i.e., the initial values of coordinates and velocities.

Key words. Stability, assymtotic stability, multidimensionality, multi-connectedness, diversity, interconnection of subsystems, Lyapunov function, multi-criteriality

INTRODUCTION

The task of the theory of motion stability is to establish signs that allow us to judge whether the considered motion will be stable or unstable. Since in reality irritating factors always inevitably exist, it becomes clear that the problem of the stability of motion acquires a very important theoretical and practical significance.

Mathematical modeling of processes and phenomena in living and non-living nature always presupposes their classification according to their complexity. Many processes and phenomena are modeled by large-scale systems (LSS), which consist of separate subsystems, united by the functions of connections. In many cases, LSS is characterized by certain features described by A.I.Kukhtenko.

These include: multidimensionality; the diversity of system structure (network, hierarchical structures, etc.);

the multi-connectedness of system elements (interconnection of subsystems at one level and communication at different levels of the hierarchy);

the diversity of the nature of elements (machines, automata, robots, etc.);

the multiplicity of changes in the composition and state of the system (variability of the structure, connections and composition of the system);

the multi-criteriality of local criteria and the global criterion for the subsystem Systems with such properties in most practically important cases are difficult to study the dynamic properties of their solutions.

For example, the study of the stability of solutions using the Lyapunov function method can be effectively carried out if there is an algorithm for constructing the Lyapunov function system with special properties. A number of research directions relate to the simplification of the original system, so that the property of stability (instability) is established not directly, but by studying the intermediate system.

Lyapunov's vector functions were most commonly used in studying the stability of LSS. It is known that for these systems, it is difficult to use classical analytical and qualitative methods for studying the dynamic properties of solutions. One of the possible approaches to the analysis of KMS is the decomposition of the original system into relatively simple subsystems, taking into account the structural features of the complex system. Lyapunov scalar functions are constructed for subsystems of a decomposed complex system, which in turn are components of the Lyapunov vector function.

Methodology: The construction of a scalar function based on a matrix function was proposed and developed in the works [8-13,16,17,18]. In this method, a scalar function is obtained by scalar multiplication of the matrix function by some constant vector to the right and left.

Modern automatic control systems are, in most cases, very complex electromechanical devices, consisting of a control object and a regulator. The purpose of the regulator is to continuously maintain some fixed or changing state in the regulating object according to a given law. Despite the efforts of many specialists in the study of dynamic properties, including stability, KMS, in most practically important cases, a number of difficulties arise. Therefore, development remains a pressing issue

a) analysis of the structure of large-scale systems,

b) obtaining the stability (asymptotic stability) and instability of the equilibrium state of large-scale systems.

To study the stability of KMS, we first introduced the concept of generalized matrix transposition and, with the help of this concept, investigated the structure of the system. Then, using the Lyapunov matrix-functions method, a sufficient condition for the stability of the equilibrium state of the linear KMS was obtained.

The work introduces the concept of generalized matrix transposition and based on this concept, various types of symmetric matrices are studied. An analysis of the structure of linear large-scale systems with different symmetry matrices was conducted.

1. Generalized transposition of matrices. It is known that the concept of matrix is widely used in the theory of differential equations, including in the theory of motion stability. Therefore, different concepts related to matrices are essential. [2, 4, 6, 7, 14, 15]

The abstract concept of the "structure" of the system [18] allowed for the formation of the concept of cohesive stability [3], which plays a significant role in the theory of KMS stability. The concept of generalized transposition and the study of the properties of matrices subjected to generalized transposition allow us to develop the theory of the stability of linear CMEs based on new forms of decomposition of the original system depending on its internal structure. The presented results illustrate the possibilities of this approach.

Results: This chapter describes the concept of generalized matrix transposition, matrix transposition with respect to the main (non-main) diagonal, vertical (horizontal) axis of symmetry and the center of the matrix. 13 properties of transposed matrices have been proven. The geometric and mechanical meaning of matrix transposition is discussed. The concepts of symmetric, oblique-symmetric and orthogonal matrices with respect to the main (non-main) diagonal, the vertical (horizontal) axis of symmetry and the center of the matrix, as well as some properties of such matrices are described. The decompositions of linear large-scale systems are considered in the case where the corresponding matrix is symmetric (closysymmetric) with respect to the main (non-main) diagonal, the vertical (horizontal) axis of symmetry and the center of the matrix.

Definition 1. The transposition of an arbitrary matrix A is called the transposition of its elements according to certain laws or rules.

Let's consider a right-angle matrix $A=(a_{ij})$ size mxn, (m$\leq$n).

The following are the elementary permutations of the arbitrary matrix elements:

1) repositioning the rows (columns) with the columns (rows) of the matrix in direct order;

2) rearranging the rows (columns) with the columns (rows) of the matrix in reverse order;

3) replacement of the i-th row with (m+1-i) -th rows, i=1,2,..,m;

4) permutation of the j-th column with (n+1-j) -th column, j=1,2,..,n;

5) the replacement of the i-th row with (m+1-i) -stars and the j-th column with (n+1-j) -stars, i=1,2,...,m; j=1,2,...,n.

Now let's formulate the definition of elementary transpositions of A matrix.

Definition 2. Matrix transposed with respect to: $A=[a_{ij}]$, $(i=1,2,...\text{m},\ j=1,2,...,n)$

1) the main diagonal (which we denote as $A^T=(a_{ji}),\ j=1,2,...,n,\ i=1,2,...,m,)$, if it is obtained by replacing the rows (columns) of matrix A with columns (rows) in the correct order;

2) non-major diagonal $A^{\perp}=(a_{n+1-j,m+1-i}),\ j=1,2,...n,\ i=1,2,...,m$, if it is obtained by replacing the rows (columns) with the columns (rows) of the A matrix in reverse order;

3) оси of the horizontal axis $A^{-}=(a_{m+1-i,j}),\ i=1,2,..,m,\ j=1,2,...,n,$ if it is obtained by replacing the i-th row with the m+1-i -th rows of the A matrix;

4) of the vertical axis $A^{|}=(a_{i,n+1-j}),\ i=1,2,...m,\ j=1,2,...,n$, if it is obtained by replacing the j-th column with n+1-j-th columns of the A matrix;

5) center $A^{0}=(a_{m+1-i,n+1-j}),\ i=1,2,...,m,\ j=1,2,...,n,$ if it is obtained by replacing the i-th row with m+1-i-th rows and the j-th column with n+1-j columns of the A matrix.

It is not difficult to verify the correctness of the following properties for the given transposed matrices by direct verification:

1. 1. If A and B are rectangular matrices of measure, then

$$(A+B)^T = A^T + B^T, \quad (A+B)^{\perp} = A^{\perp} + B^{\perp}, \quad (A+B)^{|} = A^{|} + B^{|},$$
$$(A+B)^{-} = A^{-} + B^{-}, \quad (A+B)^0 = A^0 + B^0$$

Indeed, let $A=(a_{ij}),\ B=(b_{ij})$,

$$A+B=(c_{ij})=(a_{ij}+b_{ij}), \quad i=\overline{1,m}, \quad j=\overline{1,n}$$

Then we get that

$$(A+B)^T=(c_{ij})^T=(c_{ji})=(a_{ji}+b_{ji})=(a_{ji})+(b_{ji})=(a_{ij})^T+(b_{ij})^T=A^T+B^T;$$

$$(A+B)^{\perp}=(c_{ij})^{\perp}=(c_{n+1-j,m+1-i})=(a_{n+1-j,m+1-i}+b_{n+1-j,m+1-i})=(a_{n+1-j,m+1-i})+(b_{n+1-j,m+1-i})=$$
$$(a_{ij})^{\perp}+(b_{ij})^{\perp}=A^{\perp}+B^{\perp};$$

$$(A+B)^{-}=(c_{ij})^{-}=(c_{n+1-j,i})=(a_{n+1-j,i}+b_{n+1-j,i})=(a_{n+1-j,i})+(b_{n+1-j,i})=(a_{ij})^{-}+(b_{ij})^{-}=A^{-}+B^{-};$$
$$(A+B)^{|}=(c_{ij})^{|}=(c_{i,m+1-j})=(a_{i,m+1-j}+b_{i,m+1-j})=(a_{i,m+1-j})+(b_{i,m+1-j})=(a_{ij})^{|}+(b_{ij})^{|}=A^{|}+B^{|};$$
$$(A+B)^0=(c_{ij})^0=(c_{m+1-i,n+1-j})=(a_{m+1-i,n+1-j}+b_{m+1-i,n+1-j})=(a_{m+1-i,n+1-j})+(b_{m+1-i,n+1-j})=$$
$$(a_{ij})^0+(b_{ij})^0=A^0+B^0;$$

2. If matrix A is a rectangular matrix of real number size $m \times n,\ \alpha \neq 0$, than

$$(\alpha A)^T=\alpha A^T, \ (\alpha A)^{\perp}=\alpha A^{\perp}, \ (\alpha A)^{!}=\alpha A^{!}, \ (\alpha A)^{-}=\alpha A^{-}, \ (\alpha A)^0=\alpha A^0.$$

Let $\mathrm{A}=(\mathrm{a}_{ij}),\ \mathrm{i}=1,2,\ldots,\mathrm{m},\ \mathrm{j}=1,2,\ldots,\mathrm{n},\ \alpha \neq 0 \in \mathrm{R}$, we get the following:

$$(\alpha A)^T=(\alpha a_{ij})^T=(\alpha a_{ji})=\alpha(a_{ji})=\alpha A^T$$

$$(\alpha A)^{\perp}=(\alpha a_{ij})^{\perp}=(\alpha a_{n+1-j,m+1-i})=\alpha(a_{n+1-j,m+1-i})=\alpha A^{\perp}$$

$$(\alpha A)^{-}=(\alpha a_{ij})^{-}=(\alpha a_{m+1-i,j})=\alpha(a_{m+1-i,j})=\alpha A^{-}$$

$$(\alpha A)^{|}=(\alpha a_{ij})^{|}=(\alpha a_{i,n+1-j})=\alpha(a_{i,n+1-j})=\alpha A^{|}$$

$$(\alpha A)^0=(\alpha a_{ij})^0=(\alpha a_{m+1-i,n+1-j})=\alpha(a_{m+1-i,n+1-j})=\alpha A^0$$

3. If matrix A is a rectangular matrix of real number size $m \times n$, than

$$(A^T)^T=A, \ (A^{\perp})^{\perp}=A, \ (A^{|})^{|}=A, \ (A^{-})^{-}=A, \ (A^0)^0=A.$$

Let $\mathrm{A}=(\mathrm{a}_{ij}),\ \mathrm{i}=1,2,\ldots,\mathrm{m},\ \mathrm{j}=1,2,\ldots,\mathrm{n}$ we get the following:

$$(A^T)^T=((a_{ij})^T)^T=(a_{ji})^T=(a_{ij})=A,$$

$$(A^{\perp})^{\perp}=((a_{ij})^{\perp})^{\perp}=(a_{n+1-j,m+1-i})^{\perp}=(a_{m+1-(m+1-i),n+1-(n+1-j)})=(a_{ij})=A,$$

$$(A^{-})^{-}=((a_{ij})^{-})^{-}=(a_{m+1-i,j})^{-}=(a_{m+1-(m+1-i),j})=(a_{ij})=A,$$

$$(A^{|})^{|}=((a_{ij})^{|})^{|}=(a_{i,n+1-j})^{|}=(a_{i,n+1-(n+1-j)})=(a_{ij})=A,$$

$$(A^0)^0=((a_{ij})^0)^0=(a_{m+1-i,n+1-j})^{|}=(a_{m+1-(m+1-i),n+1-(n+1-j)})=(a_{ij})=A.$$

4. If A and B are rectangular size matrices $m \times n$ и $n \times k$, than

$$(AB)^T = B^T A^T,\quad (AB)^{\perp} = B^{\perp} A^{\perp},\quad (AB)^0 = A^0 B^0.$$

Let

$A = (a_{ij}),\ i=1,2,\dots,m,\ \ j=1,2,\dots,n;\ \ B=(b_{jl}),\ i=1,2,\dots,n,\ \ l=1,2,\dots,k$.

Than we have $AB=(c_{il}), i=1,2,..,m, l=1,2,\dots,k$

here $(c_{il})=\sum_{j=1}^{n} a_{ij}b_{jl}$ and we get the following equations:

$$(AB)^T = (c_{il})^T = (c_{li}) = \left(\sum_{j=1}^{n} b_{lj}a_{ji}\right) = (b_{lj})(a_{ji}) = B^T A^T,$$

$$(AB)^{\perp} = (c_{il})^{\perp} = (c_{k+1-l,m+1-i}) = \left(\sum_{j=1}^{n} b_{k+1-l,j}a_{jm+1-i}\right) = (b_{k+1-l,j})(a_{j,m+1-i}) = B^{\perp}A^{\perp},$$

$$(AB)^{-} = (c_{il})^{-} = (c_{m+1-i,l}) = \left(\sum_{j=1}^{n} a_{m+1-i,j}b_{j,l}\right) = A^{-}B,\qquad A^{-} = (a_{m+1-i,j}),$$

$$(AB)^{|} = (c_{il})^{|} = (c_{i,k+1-l}) = \left(\sum_{j=1}^{n} a_{ij}b_{j,k+1-l}\right) = AB^{|},\qquad B^{|} = (b_{j,k+1-l}),$$

$$(AB)^0 = (c_{il})^0 = (c_{m+1-i,k+1-l}) = \left(\sum_{j=1}^{n} a_{m+1-i,j}b_{j,k+1-l}\right) = (a_{m+1-i,j})(b_{j,k+1-l}) = A^0B^0,$$

5. If A is an n-square matrix, then

$$(A^T)^{\perp} = (A^{\perp})^T = A^0,\ \ (A^{|})^{-} = (A^{-})^{|} = A^0,\ \ (A^0)^T = (A^T)^0 = A^{\perp},$$
$$(A^0)^{\perp} = (A^{\perp})^0 = A^T,\ \ (A^0)^{|} = (A^{|})^0 = A^{-},\ \ (A^0)^{-} = (A^{-})^0 = A^{|}.$$

Indeed,

$$(A^T)^{\perp} = ((a_{ij})^T)^{\perp} = (a_{ji})^{\perp} = (a_{n+1-i,n+1-j}) = A^0;$$

$$(A^{\perp})^T = ((a_{ij})^{\perp})^T = (a_{n+1-j,n+1-i})^T = (a_{n+1-i,n+1-j}) = A^0;$$

$$(A^{|})^{-} = ((a_{ij})^{|})^{-} = (a_{n+1-i,j})^{-} = (a_{n+1-i,n+1-j}) = A^0;$$

$$(A^{-})^{|} = ((a_{ij})^{-})^{|} = (a_{i,n+1-j})^{|} = (a_{n+1-i,n+1-j}) = A^0;$$

$$(A^0)^T = ((a_{ij})^0)^T = (a_{n+1-i,n+1-j})^T = (a_{n+1-j,n+1-i}) = A^{\perp};$$

$$(A^T)^0 = ((a_{ij})^T)^0 = (a_{ji})^0 = (a_{n+1-j,n+1-i}) = A^{\perp};$$

$$(A^0)^{\perp} = ((a_{ij})^0)^{\perp} = (a_{n+1-i,n+1-j})^{\perp} = (a_{n+1-(n+1-j),n+1-(n+1-i)}) = (a_{ji}) = A^T;$$

$$(A^{\perp})^0 = ((a_{ij})^{\perp})^0 = (a_{n+1-j,n+1-i})^0 = (a_{n+1-(n+1-j),n+1-(n+1-i)}) = (a_{ji}) = A^T;$$

$$(A^0)^{|} = ((a_{ij})^0)^{|} = (a_{n+1-i,n+1-j})^{|} = (a_{n+1-i,n+1-(n+1-j)}) = (a_{n+1-i,j}) = A^{-};$$

$$(A^{|})^0 = ((a_{ij})^{|})^0 = (a_{i,n+1-j})^0 = (a_{n+1-i,n+1-(n+1-j)}) = (a_{n+1-i,j}) = A^{-};$$

$$(A^0)^{-} = ((a_{ij})^0)^{-} = (a_{n+1-i,n+1-j})^{-} = (a_{n+1-(n+1-i),n+1-j}) = (a_{i,n+1-j}) = A^{|};$$

$$(A^{-})^0 = ((a_{ij})^{-})^0 = (a_{n+1-i,j})^0 = (a_{n+1-(n+1-i),n+1-j}) = (a_{i,n+1-j}) = A^{|}.$$

6. If A is an n-square matrix, then

$$A=(A^0)^{-1}(A^TA^{\perp})^T=(A^0)^{-1}(A^{\perp}A^T)^{\perp}=(A^{\perp}A^T)^T(A^0)^{-1}=(A^TA^{\perp})^{\perp}(A^0)^{-1}.$$

Indeed,

$$(A^0)^{-1}(A^TA^{\perp})^T=(A^0)^{-1}(A^{\perp})^T(A^T)^T=(A^0)^{-1}A^0A=A;$$

$$(A^0)^{-1}(A^{\perp}A^T)^{\perp}=(A^0)^{-1}(A^T)^{\perp}(A^{\perp})^{\perp}=(A^0)^{-1}A^0A=A;$$

$$(A^{\perp}A^T)^T(A^0)^{-1}=(A^T)^T(A^{\perp})^T(A^0)^{-1}=AA^0(A^0)^{-1}=A;$$

$$(A^TA^{\perp})^{\perp}(A^0)^{-1}=(A^{\perp})^{\perp}(A^T)^{\perp}(A^0)^{-1}=AA^0(A^0)^{-1}=A.$$

7. If A is an n-square matrix, then

$$\left|A^T\right|=\left|A^{\perp}\right|=\left|A^0\right|=|A|,\quad \left|A^{|}\right|=\left|A^{-}\right|=(-1)^{\alpha}|A|,$$

here α - the number of columns or rows of a matrix A that are moved to obtain from A a matrix $A^{|}$ or A^{-}.

The validity of these equations follows from the definition 1, the definition 2 and the properties of the determinant.

8. If A is a square matrix, then the following equalities hold:

1) $E'A=E^{-}A=A^{-}$, 2) $AE'=AE^{-}=A'$, 3) $E'AE'=E^{-}AE^{-}=A^0$,
4) $A^TE'=A'$, $E'A^T=A^{-}$, 5) $A^{\perp}E'=A^{-}$, $E'A^{\perp}=A'$,
6) $A^{-}E'=A^0$, $E'A^{-}=A$, 7) $A'E'=A$,

where E is the identity matrix

$$E'=E^{-}=\begin{pmatrix} 0\ 0 \ldots 1 \\ \ldots\ldots\ \ \ldots\ldots \\ 1\ 0 \ldots\ldots 0\end{pmatrix}.$$

In fact, let E be an n-th order unit matrix and

$$A=(a_{ij}),\quad (i,j=1,2,\ldots,n)\ ,$$

then we have

$$E'A=\begin{pmatrix} 0 & 0 & \ldots & 0 & 1 \\ 0 & 0 & \ldots & 1 & 0 \\ \ldots & & \ldots & \ldots & \ldots \\ 1 & 0 & \ldots & 0 & 0 \end{pmatrix}\begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{pmatrix}=\begin{pmatrix} a_{n1} & a_{n2} & \ldots & a_{nn} \\ a_{n-1,1} & a_{n-1,2} & \ldots & a_{n-1,n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{11} & a_{12} & \ldots & a_{1n} \end{pmatrix}=A^{-}.$$

Similarly, the validity of the remaining equalities is checked.

9. If A is an odd square matrix of order n, then

$$(A^{-1})^T=(A^T)^{-1},\ (A^{-1})^{\perp}=(A^{\perp})^{-1},\ (A^{-1})^{|}=(A^{|})^{-1},\ (A^{-1})^{-}=(A^{-})^{-1},\ (A^{-1})^0=(A^0)^{-1}.$$

Indeed, let $A=(a_{ij}),\ i,j=1,2,\ldots,n,$ then $A^{-1}=\dfrac{1}{|A|}(A_{ji}),$

here A_{ij} - algebraic additions to an element a_{ij}, $|A|=\det A.$

$$(A^T)^{-1}=((a_{ij})^T)^{-1}=\left(a_{ji}\right)^{-1}=\frac{1}{|A^T|}(A_{ij})=\frac{1}{|A|}(A_{ij})=\frac{1}{|A|}(A_{ji})^T=\left(\frac{1}{|A|}(A_{ij})\right)^T=(A^{-1})^T;$$

$$(A^{\perp})^{-1}=((a_{ij})^{\perp})^{-1}=(a_{n+1-j,n+1-i})^{-1}=\frac{1}{|A^{\perp}|}(A_{n+1-i,n+1-j})=\frac{1}{|A|}(A_{ji})^{\perp}=\left(\frac{1}{|A|}(A_{ji})\right)^{\perp}=(A^{-1})^{\perp};$$

$$(A^{|})^{-1}=((a_{ij})^{|})^{-1}=(a_{i,n+1-j})^{-1}=\frac{1}{|A^{|}|}(A_{n+1-j,i})=\frac{1}{(-1)^{\alpha}|A|}(A_{ji})^{|}=\left(\frac{1}{(-1)^{\alpha}|A|}(A_{ji})\right)^{|}=(A^{-1})^{|};$$

$$(A^{-})^{-1}=((a_{ij})^{-})^{-1}=(a_{n+1-i,j})^{-1}=\frac{1}{|A^{-}|}(A_{j,n+1-i})=\frac{1}{(-1)^{\alpha}|A|}(A_{ji})^{-}=\left(\frac{1}{(-1)^{\alpha}|A|}(A_{ji})\right)^{-}=(A^{-1})^{-};$$

$$(A^{0})^{-1}=((a_{ij})^{0})^{-1}=(a_{n+1-i,n+1-j})^{-1}=\frac{1}{|A^{0}|}(A_{n+1-j,n+1-i})=\frac{1}{|A|}(A_{ji})^{0}=\left(\frac{1}{|A|}(A_{ji})\right)^{0}=(A^{-1})^{0}.$$

10. If A is an n-th order quadratic matrix, E is an n-th order unit matrix, and λ is a parameter, then

$$|A^T-\lambda E|=|A^{\perp}-\lambda E|=|A^{0}-\lambda E|=|A-\lambda E|.$$

The validity of these equalities follows from the properties of 1, 2, 7, and

$$E=E^T=E^{\perp}=E^{0}.$$

11. $Sp(A^T)=Sp(A^{\perp})=Sp(A^{0})=Sp(A),$ here Sp - trace matrix.

Indeed, when transposing across the main diagonal and the non-main diagonal and relative to the center of the matrix, all the elements of the main diagonal remain on the same diagonal.

12. $rang(A^T)=rang(A^{\perp})=rang(A^{!})=rang(A^{-1})=rang(A^{0})=rang(A)$,

here $rang(A)$ - matrix rank A.

The validity of these equations follows from the property 9 and the definition of the rank of the matrix.

13. Let A be a square matrix, $\Delta_i,\Delta_i^T,\Delta_i^{\perp},\Delta_i^{0}$ $(\overline{\Delta}_i,\overline{\Delta}_i^{T},\overline{\Delta}_i^{\perp},\overline{\Delta}_i^{0}),(i=1,2,...,n)-$ the main minor (corresponding to them additional minor) of the matrix $A, A^T, A^{\perp}, A^{0}$, respectively.

Then the following equations are true:

1) $\Delta_i=\overline{\Delta}_{n-i}^{0},\ i=1,2,...,n-1,\ \Delta_n=\Delta_n^{0}=|A|=|A^{0}|,$

2) $\Delta_i^{0}=\overline{\Delta}_{n-i},\ i=1,2,...,n-1,$

3) $\Delta_i=\Delta_i^{T},\ i=1,2,...,n,\ \Delta_n=\Delta_n^{T}=|A|=|A^{T}|,$

4) $\Delta_i=\overline{\Delta}_{n-i}^{\perp},\ i=1,2,...,n-1,\ \Delta_n=\Delta_n^{\perp}=|A|=|A^{\perp}|.$

The validity of these equations follows from the definition of matrix transposition, the properties of 7, 9 and the properties of the determinant.

Example 1. Let A be a matrix of the form

$$A=(a_{ij}),\ i=1,2,3,\quad j=1,2,3,4.$$

Then we have

$$A^{T}=\begin{pmatrix} a_{11} & a_{21} & a_{31}\\ a_{12} & a_{22} & a_{32}\\ a_{13} & a_{23} & a_{33}\\ a_{14} & a_{24} & a_{44}\end{pmatrix},\quad A^{\perp}=\begin{pmatrix} a_{34} & a_{24} & a_{14}\\ a_{33} & a_{23} & a_{13}\\ a_{32} & a_{22} & a_{12}\\ a_{31} & a_{21} & a_{11}\end{pmatrix}.$$

$$A^{|}=\begin{pmatrix} a_{14} & a_{13} & a_{12} & a_{11}\\ a_{24} & a_{23} & a_{22} & a_{21}\\ a_{34} & a_{33} & a_{32} & a_{31}\end{pmatrix},\quad A^{-}=\begin{pmatrix} a_{31} & a_{32} & a_{33} & a_{34}\\ a_{21} & a_{22} & a_{23} & a_{24}\\ a_{11} & a_{12} & a_{13} & a_{14}\end{pmatrix},\quad A^{0}=\begin{pmatrix} a_{34} & a_{33} & a_{32} & a_{31}\\ a_{24} & a_{23} & a_{22} & a_{21}\\ a_{14} & a_{13} & a_{12} & a_{11}\end{pmatrix},$$

2. The geometric and mechanical meaning of matrix transposition.

It is known that each rectangular matrix can be compared to some rectangle, which contains all the elements of the matrix under consideration.

The major (non-major) diagonal of matrix A is a segment of the line passing through points containing elements a_{ii}, $i=1,2,...,m$ ($a_{i,m+1-i}$, $i=1,2,....,m$).

The vertical (horizontal) axis of a matrix A is called the vertical (horizontal) axis of symmetry of a rectangle.

The center of a matrix is called the center of symmetry of a rectangle. Note that the principal (non-main) diagonal of the matrix A does not coincide with the diagonal of the rectangle. Therefore, when transposing, the size of the matrix changes to nxm.

If m = n, then the main (non-main) diagonal of the matrix A coincides with the left (right) diagonal of the square.

Thus, from a geometric point of view, matrix transposition is performed relative to a point or a line. If the point (a line) is the center (axis) of symmetry of the rectangle in which the matrix lies, then the dimension of the transposed matrix does not change.

The elements of the matrix located in the point or on the line with respect to which the transposition is performed remain unchanged. A transposed matrix corresponds to a rectangle rotated 1800 around a point or a line relative to which transposition is performed.

Example 2. Let A be a matrix of the form. $A=(a_{ij}),\ i,j=1,2,3.$

Than we have

$$A^{T}=\begin{pmatrix} a_{11} & a_{21} & a_{31}\\ a_{12} & a_{22} & a_{32}\\ a_{13} & a_{23} & a_{33}\end{pmatrix},\quad A^{\perp}=\begin{pmatrix} a_{33} & a_{32} & a_{31}\\ a_{23} & a_{22} & a_{21}\\ a_{13} & a_{12} & a_{11}\end{pmatrix},\quad A^{|}=\begin{pmatrix} a_{13} & a_{12} & a_{11}\\ a_{23} & a_{22} & a_{21}\\ a_{33} & a_{32} & a_{31}\end{pmatrix},$$

$$A^{-}=\begin{pmatrix} a_{31} & a_{32} & a_{33}\\ a_{21} & a_{22} & a_{23}\\ a_{11} & a_{12} & a_{13}\end{pmatrix},\qquad A^{0}=\begin{pmatrix} a_{33} & a_{32} & a_{31}\\ a_{23} & a_{22} & a_{21}\\ a_{13} & a_{12} & a_{11}\end{pmatrix}.$$

Note that element a_{22} retains its position for all transpositions, moreover, when transposed relative to the main (non-main) diagonal, elements a_{11}, a_{33} (a_{13}, a_{31}) relative to the horizontal (vertical) axis of symmetry remain unchanged, and when transposed relative to the center of the matrix, all elements a_{21}, a_{23} (a_{12}, a_{32}) of the A matrix, except a_{22}, change with the corresponding elements.

To reveal the connection between the generalized transposition of matrices and the structure of the linear LSS, we will establish the following correspondences:

Let A = (a_{ij}) be a rectangular matrix of dimension mxn (for $m \leq n$ certainty) and LSS in R^n consists of m free subsistence. Free subsistences correspond to the elements of the main diagonal of the matrix a_{ii}, $(i = 1, 2, ..., m)$ in such a way that the subsistence corresponding to the element a_{ii}, $(i = 1, 2, ..., m)$ is balanced with the subsistence corresponding to the element $a_{m+1-i, m+1-i}$. In this case, the a_{ij}, $(i, j = 1, 2, ..., m,\ i < j\,(i > j))$ elements correspond to the connections (reverse connections) between the free subsystems a_{ii} and a_{jj}, $(i, j = 1, 2, .., m)$, i.e., we have the influence of the subsystem a_{ii} (a_{jj}) on the subsystem corresponding to a_{jj} (a_{ii}). We will call these bonds the internal bonds of LSS. The remaining elements a_{ij}, $(i = 1, 2, ...m,\ j = m+1, ...n)$ i.e., the elements, correspond to the connections of the free subsystems with other subsystems, which act together with the given LSS. We will call these connections external connections. All connections are internal.

It should be noted that for such a consistency of elements in the non-main diagonal, the balancing subsistence connections correspond. If m is even, then for each free subsystem there exists a balancing subsystem. If m is odd, then one subsystem corresponding to element $a_{\frac{m+1}{2}, \frac{m+1}{2}}$ does not have a balancing subsystem. Therefore, we will call this subsystem the reference one and consider it separately (for example, in a large-scale energy system, one machine is usually considered as the reference one).

Therefore, the transposition process describes the change in the internal structure of the LSS

From a geometric point of view, the meaning of this definition is that the rotation of the corresponding rectangle by 180 occurs around a line passing through:

1) the principal diagonal of the matrix A;

2) non-major diagonal of matrix A;

3) the horizontal axis of the matrix A;

4) the vertical axis of the matrix A;

5) the center of the matrix A

accordingly.

Applied to CMC, the meaning of generalized transposition lies in adequately describing the internal structure of CMC under certain physical changes. At the same time, the structure of CMC can be changed in such a way that:

− without changing free subsystems, only connections between free subsystems with corresponding feedback are transformed;

− without changing the connections, the feedback between the balanced pairs of free subsistences is transformed, or the balancing subsistences are transformed between themselves and the connections (reverse connections), except for the connections (reverse connections) between the balancing subsistence pairs;

− free subsystems with feedback and connections corresponding to pairs of balancing free subsystems are transformed in reverse order;

− free subsystems with connections and feedback between the corresponding pairs of balancing free subsystems are transformed in direct order;

− the corresponding balancing free pairs of subsystems are completely transformed between themselves and their connections with feedback, except for the reference subsystem (if there is one).

Conclusion.

If n (m) is odd, then when transposing relative to the vertical (horizontal) axis, the reference subsystem does not change, and accordingly, the vertical (horizontal) connections associated with these subsystems do not change. If n (m) is even, then such a subsystem does not exist.

2. If n and m are odd, then when transposing the A matrix relative to the center in the KMS structure, only the reference subsystem does not change. If m or n are even, then such a subsystem does not exist.

3. Structures of KMS and properties of the symmetric matrix.

Consider the square nxn matrix

$$A=(a_{ij}),\ i,j=1,2,...,n,$$

describing some linear large-scale system.

Definition 3. An A matrix is called generalized symmetric if for each of its elements there exists an element that is symmetric with respect to some point or elements of this matrix on a line.

The elements of a matrix lying at this point or on a line are considered to be symmetrical to themselves.

Next, we will describe all types of symmetric matrices.

Let A be a square matrix, then:

− the left (right) diagonal of the square is called the main (non-main) diagonal of the matrix A;

− the vertical (horizontal) axis of symmetry of the square is called the vertical (horizontal) axis of the matrix A respectively;

- the center of symmetry of the square is called the center of the matrix A.

Definition 4. The square nxn matrix A is called symmetric with respect to:

− the main diagonal, if $A^T=A$, т.е. $a_{ij}=a_{ji},\ i,j=1,2,...,n$, (ordinary symmetric matrix);

− non-major diagonal, if $A^{\perp}=A$, т.е. $a_{ij}=a_{n+1-j,n+1-i},\ i,j=1,2,...,n$;

− the vertical axis, if $A^{!}=A$, т.е. $a_{ij}=a_{i,n+1-j},\ i,j=1,2,...,n$;

– the horizontal axis, if $A^{-} = A$, т.е. $a_{ij} = a_{n+1-i,j}$, $i, j = 1, 2, ..., n$;

– matrix center, if $A^{0} = A$, т.е. $a_{ij} = a_{n+1-i,n+1-j}$, $i, j = 1, 2, ..., n$.

Note that the main and non-main diagonals contain the elements of the matrix A, and the vertical and horizontal axes contain the elements of the matrix A for even n, and the elements of the matrix A for odd n. In the center of the matrix, for even n, there is no, and for odd n, there is an element $a_{\frac{n+1}{2},\frac{n+1}{2}}$.

Definition 5. KMS in R^n, consisting of n free subsystems, is called symmetric with respect to:

– free subsystems, if the corresponding connections and feedback are the same;

- connections and feedback between the balancing pairs of free subsystems, if the balancing subsystems are the same between themselves and the connections, except for the connections between the balancing subsystems, with corresponding feedback;

– KMS center, if the balancing subsystems with their pairs and all connections with corresponding feedback are identical.

Example 3. Many oscillatory systems are systems of two or more connected oscillators. [1,3] Examples include molecules (atoms interacting with each other), pendulums oscillating around the same axis (they are connected by elastic forces in the axis), and connected electrical circuits. The characteristics of oscillations in such systems will be considered on the example of two mathematical or physical pendulums connected by a spring (Fig. 1).

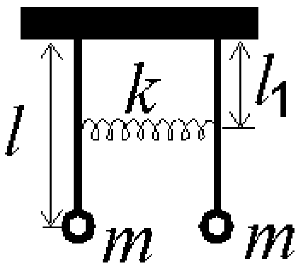

Figure 1. Two pendulums connected by a spring.

It is known that a spherical pendulum has two degrees of freedom, that is, to describe its motion, two parameters are required - displacement angles in two mutually perpendicular planes.

A system of two pendulums is described by four parameters and thus has four degrees of freedom. If the oscillations corresponding to each degree of freedom are independent, then the problem of describing motion is purely kinematic, i.e. the problem of decomposing a complex motion into a sum of simpler motions. If there is a dynamic relationship between the movements of different degrees of freedom, in which excitation of one degree of freedom causes dynamic changes in all other degrees of freedom, then this leads to the exchange of vibrational energy between degrees of freedom, leading to new physical phenomena that are absent in the system of independent pendulums.

It is known that for a free fluid the equation of moments is

$$J\ddot{\alpha} = mgl\sin\alpha\,, \tag{1}$$

where J is the moment of inertia of the pendulum, m, l is its mass and length respectively, α is the angle of deviation from the equilibrium position.

In the case of two pendulums connected by a spring (Fig. 1), an additional force from spring $F_{св}$ will be applied to each pendulum, which with small deviations can be determined from Hooke's law

$$F_{св} = kl_1(\alpha_2 - \alpha_1), \tag{2}$$

where l_1-is the distance from the spring attachment point to the spring attachment point. This force creates an additional moment acting on each of the pendulums. In this case, the equations of motion of the pendulums will have the form:

$$\begin{aligned} ml^2\ddot{\alpha}_1 &= -mgl\sin\alpha_1 + F_{св}l_1\cos\alpha_1 \\ ml^2\ddot{\alpha}_2 &= -mgl\sin\alpha_2 + F_{св}l_1\cos\alpha_2 \end{aligned}, \tag{3}$$

where $J = ml^2$. Linearizing the system (1.3) and taking into account (1.2) we get

$$\begin{aligned} \ddot{\alpha}_1 &= -\left(\frac{g}{l} + \frac{kl_1^2}{ml^2}\right)\alpha_1 + \frac{kl_1^2}{ml^2}\alpha_2 \\ \ddot{\alpha}_2 &= \frac{kl_1^2}{ml^2}\alpha_1 - \left(\frac{g}{l} + \frac{kl_1^2}{ml^2}\right)\alpha_2 \end{aligned}, \tag{4}$$

where α_1, α_2- deviation of the pendulum from the equilibrium position, $\omega_0 = \frac{g}{l} + \frac{kl_1^2}{ml^2}$- eigenfrequencies of the pendulum (particle frequencies) $\mu = \frac{kl_1^2}{ml^2}$ - the coefficients that determine the magnitude of the connection between the pendulums.

By replacing variables $\alpha_1 = x_1,\ \dot{\alpha}_1 = x_2,\ \ \dot{\alpha}_2 = x_3,\ \alpha_2 = x_4$ The system (4) is reduced to the following equations:

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -\omega_0 x_1 + \mu x_4, \\ \dot{x}_3 &= -\omega_0 x_4 + \mu x_1, \\ \dot{x}_4 &= x_3, \end{aligned} \tag{5}$$

or in matrix form

$$\dot{x} = Ax, \tag{6}$$

where $x^T = (x_1, x_2, x_3, x_4)$.

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\omega_0 & 0 & 0 & \mu \\ \mu & 0 & 0 & -\omega_0 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

Thus, for the case under consideration, the system (5) corresponds to an A matrix symmetrical to the center of the matrix.

The decomposition of the system (6) relative to the horizontal and vertical axes of symmetry

$$\dot{y}=\begin{pmatrix}0 & 1\\ -\omega_0 & 0\end{pmatrix}y+\begin{pmatrix}0 & 0\\ 0 & \mu\end{pmatrix}z, \qquad \dot{z}=\begin{pmatrix}\mu & 0\\ 0 & 0\end{pmatrix}y+\begin{pmatrix}0 & -\omega_0\\ 1 & 0\end{pmatrix}z,$$

where $y^T=(x_1,x_2)$, $z^T=(x_3,x_4)$, $x^T=(y^T,\,z^T)$, shows that the equations correspond to the free subsystems

$$\dot{y}=\begin{pmatrix}0 & 1\\ -\omega_0 & 0\end{pmatrix}y=A_1y, \qquad \dot{z}=\begin{pmatrix}0 & -\omega_0\\ 1 & 0\end{pmatrix}z=A_1^0\,z.$$

It is not difficult to verify that the equilibrium state of system (6) is stable (but not asymptotically) under the condition

$$|k|<\frac{mg\ell}{2\ell_1^2}.$$

The following statements about the properties of a symmetric matrix are valid:

1. Matrix A, symmetrical with respect to the main and non-main matrix the diagonal is symmetrical to the center,

In fact, let the n th order quadratic matrix $A=(a_{ij}),\ i,j=1,2,...,n$ be symmetric with respect to the principal and non-core diagonal, i.e. $A^T=A=A^{\perp}$ or $a_{ij}=a_{ji};\ a_{ij}=a_{n+1-j,n+1-i},\ i,j=1,2,...,n.$. It follows from this that $a_{ji}=a_{n+1-j,n+1-i}$ or $a_{ij}=a_{n+1-i,n+1-j}$.

Therefore we have

$$A=(a_{ij})=(a_{ji})=(a_{n+1-j,n+1-i})=(a_{n+1-i,n+1-j})=A^0.$$

Based on the definition (4), the matrix A is symmetric about the center of the matrix.

1. A matrix, symmetric with respect to the vertical and is symmetrical to the center of the matrix.

Indeed, let it be $A^{-}=A=A^{|}$, i.e. $a_{ij}=a_{n+1-i,j}$ and $a_{ij}=a_{i,n+1-j},\ i,j=1,2,...,n.$ It follows that

$$a_{n+1-i,j}=a_{i,n+1-j}\ \text{ or }\ a_{ij}=a_{n+1-i,n+1-j}$$

therefore $A=(a_{ij})=(a_{n+1-i,j})=(a_{i,n+1-j})=(a_{n+1-i,n+1-j})=A^0$, i.e. based on the 1-4 definition, matrix A is symmetric with respect to the center of the matrix.

1. A matrix that is symmetric with respect to a vertical axis is a special matrix.

Indeed, if A is symmetric with respect to the vertical (horizontal) axis, then, based on the definition (4), $a_{ij}=a_{i,n+1-j}$ $(a_{ij}=a_{n+1-i,j})$. From this it follows that the j and $n+1-j$ columns (i and $n+1-i$ rows) of the matrix have the same elements. It is known that the determinant of such a matrix is zero and therefore A is a special matrix.

2. For each quadratic matrix A a matrix

$$S_1=\frac{1}{2}(A+A^T),\ S_2=\frac{1}{2}(A+A^{\perp}),\ \ S_3=\frac{1}{2}(A+A^{|}),\ S_4=\frac{1}{2}(A+A^{-}),\ S_5=\frac{1}{2}(A+A^0),$$

are symmetric with respect to the main diagonal, the non-main diagonal, the vertical axis, the horizontal axis, and the center of the matrix, respectively.

Indeed, based on the properties 1, 3, we have

$$S_1^T=\frac{1}{2}(A+A^T)^T=\frac{1}{2}(A^T+A)=\frac{1}{2}(A+A^T)=S_1,$$

$$S_2^{\perp}=\frac{1}{2}(A+A^{\perp})^{\perp}=\frac{1}{2}(A^{\perp}+A)=\frac{1}{2}(A+A^{\perp})=S_2,$$

$$S_3^{|}=\frac{1}{2}(A+A^{|})^{|}=\frac{1}{2}(A^{|}+A)=\frac{1}{2}(A+A^{|})=S_3,$$

$$S_4^{-}=\frac{1}{2}(A+A^{-})^{-}=\frac{1}{2}(A^{-}+A)=\frac{1}{2}(A+A^{-})=S_4,$$

$$S_5^{0}=\frac{1}{2}(A+A^{0})^{0}=\frac{1}{2}(A^{0}+A)=\frac{1}{2}(A+A^{0})=S_5.$$

5. If A is a symmetric matrix with respect to the main (non-main) diagonal, the center of the matrix, then the matrices

$$A^i(i=1,2,\ldots),\ \ \alpha A,\ \ T^{\mathrm{T}}AT,\ \ T^{\perp}AT,$$

which are also symmetric matrices with respect to the main

(non-main) diagonal, the center of the matrix respectively, where T is an arbitrary non-specific square matrix, α is a real number,

Indeed, if

$a)$ $A^T=A,$ then

$$(A^i)^T=(A^T)^i=A^i,\ (i=1,2,\ldots),\ \ (\alpha A)^T=\alpha A^T=\alpha A,\ \ (T^TAT)^T=(AT)^T(T^T)^T=T^TA^TT=T^TAT;$$

$б)$ $A^{\perp}=A,$ then

$$(A^i)^{\perp}=(A^{\perp})^i=A^i,\ (i=1,2,\ldots),\ \ (\alpha A)^{\perp}=\alpha A^{\perp}=\alpha A,\ \ (T^{\perp}AT)^{\perp}=(AT)^{\perp}(T^{\perp})^{\perp}=T^{\perp}A^{\perp}T=T^{\perp}AT;$$

$в)$ $A^0=A,$ then $(A^i)^0=(A^0)^i=A^i,\ (i=1,2,\ldots),\ \ (\alpha A)^0=\alpha A^0=\alpha A.$

1. If the non-singular square matrix A is symmetric with respect to the main (non-main) diagonal and the center of the matrix, then A^{-1} is also symmetric with respect to the main (non-main) diagonal and the center of the matrix, respectively.

   Indeed, based on the properties of 9, if we have

   $a)$ $A^T=A,$ то $(A^{-1})^T=(A^T)^{-1}=A^{-1},$ $б)$ $A^{\perp}=A,$ то $(A^{-1})^{\perp}=(A^{\perp})^{-1}=A^{-1},$

   $в)$ $A^0=A,$ то $(A^{-1})^0=(A^0)^{-1}=A^{-1}.$

2. If the square matrices A and B are symmetric with respect to the center of the matrix, then the matrices AB and BA are also symmetric with respect to the center of the matrix.

Indeed, if $A=A^0$ and $B=B^0$, then, based on the last equality of the 4 properties, we have

$$(AB)^0=A^0B^0=AB,\ \ (BA)^0=B^0A^0=BA.$$

3. If the n-th quadratic matrix A is symmetric with respect to the center of the matrix and Δ_i, $i=1,2,\dots,n$ the principal minor of the matrix A, and the minor of $\overline{\Delta}_i$ the corresponding additional minor of the matrix A, then

$$\Delta_i = \overline{\Delta}_{n-i},\ i=1,2,\dots,n-1.$$

Proof. Let n-order quadratic matrix A, symmetrical to the center of the matrix, have the form

$$A=(a_{k,j}),\quad a_{kj}=a_{n+1-k,n+1-j},\ k,j=1,2,\dots,n\ . \tag{7}$$

Let's consider matrices

$$A_i=(a_{k,j}),\ k,j=1,2,\dots,i,\quad 1\le i\le n \tag{8}$$

$$A_i^*=(a_{n+1-k,n+1-j}),\ k,j=i,i-1,i-2,\dots,1,\ 1\le i\le n\ . \tag{9}$$

From (7) it follows that $|A_i|=|A_i^*|$. This can be confirmed by direct verification.

For example, at $i=1$, $A_1=(a_{11})$, $A_1^*=(a_{nn})$, based on (7) $a_{11}=a_{nn}$, at $i=2$, $A_2=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{pmatrix}$, $A_2^*=\begin{pmatrix} a_{n-1,n-1} & a_{n-1,n} \\ a_{n,n-1} & a_{nn}\end{pmatrix}$,

from (7), it follows that $A_2^*=A_2^0$, therefore $|A_2|=|A_2^*|$. Since for all $i\le\left[\frac{n}{2}\right]$ on the basis of (7) we have $A_i^*=A_i^0$ and for $i\ge\left[\frac{n}{2}\right]$ starting from the last row (column) of the matrix A_i coincides in reverse order with the first rows (columns) of the matrix A_i^* and finally for $i=n$ the matrices A_i and A_i^* coincide. The A_i matrices correspond to the major minor of the A matrix, i.e.

$$\Delta_i=|A_i|=|A_i^*|,\ i=1,2,\dots,n \text{ where } A_n=A.$$

From (8) it follows that the additional minor $\overline{\Delta}_i$ of the matrix A corresponds to the matrix $\overline{A}_i == (a_{i+k,i+j}),\ k,j=1,2,\dots,n-i$.

It follows that

$$\overline{A}_{n-i}=(a_{n-i+k,n-i+j}),\ k,j=1,2,\dots,i\ . \tag{10}$$

Comparing (9) and (10), we get

$$\overline{A}_{n-i}=A_i^*.$$

So, $\Delta_i=|A_i|=|A_i^*|=|\overline{A}_{n-i}|=\overline{\Delta}_{n-i}$.

4. If the square matrix A is symmetric with respect to the center of the matrix, then the conditions

$$\overline{\Delta}_i>0,\ i=1,2,\dots,n-1,\ \Delta_n=|A|>0\quad , \tag{11}$$

necessary and sufficient for the positive determination of the A matrix. For the negative determination of the A matrix, the conditions (11) take the form

$$(-1)^i\overline{\Delta}_i>0,\ i=1,2,\dots,n-1,\ (-1)^n\Delta_n=(-1)^n|A|>0\ , \tag{12}$$

if n is even,

$$(-1)^{i-1}\overline{\Delta}_i > 0,\ i=1,2,\dots,n-1,\ (-1)^n\,\Delta_n = (-1)^n\,|A| > 0,$$

if n is odd.

Proof. Let n-order quadratic matrix A be symmetric with respect to the center of the matrix.

Necessity. If the matrix A is positive-determined, then based on the Sylvester criterion $\Delta_i > 0,\ i=1,2,\dots,n-1,\ \Delta_n = |A| > 0$ and then based on the statement 8 we have

$$\Delta_i = \overline{\Delta}_{n-i} > 0, \quad i=1,2,\dots,n-1.$$

Discussion: It is known that if i takes the values of $1,2,\dots,n-1,$ then $n-i$ also takes these values in the opposite order, so it follows from $\Delta_{\mathrm{i}} = \overline{\Delta}_{n-i} > 0$ that $\Delta_{\mathrm{n-i}} = \overline{\Delta}_i > 0.$

Sufficiency. Based on the statement 8, we have

$$\Delta_i = \overline{\Delta}_{n-i},\ \ i=1,2,\dots,n-1 \text{ or } \Delta_{n-i} = \overline{\Delta}_i,\ \ i=1,2,\dots,n-1, \tag{13}$$

if $\overline{\Delta}_i > 0,\ \ i=1,2,\dots,n-1,\ \ \Delta_n = |A| > 0$, then it follows from (13) that . Since takes the values of , then also takes these values in the opposite order, so . It follows from this that . Based on the Sylvester criterion, the matrix A is positively defined.

The second part of statement 9 is also proven.

5. If the square nxn matrix A is symmetric with respect to 1) the main diagonal, 2) the non-main diagonal, 3) the vertical axis of the matrix, 4) the horizontal axis of the matrix, 5) the center of the matrix, then it can be broken down into block matrices of the form:

1) at n=2k at n=2k+1

2) at n=2k at n=2k+1

3) at n=2k at n=2k+1

4) at n=2k at n=2k+1

5) at n=2k at n=2k+1

Here, all block matrices are square k-th order

.

The validity of this statement follows from definition 2 and definition 4.

Definition 6. The square matrix of the nth order is called a co-osymmetric (anti-symmetric) with respect to:

– the main diagonal, if ;

– non-main diagonal, if ;

– the vertical axis, if ;

– horizontal axis, if ;

– the center of the matrix, if .

Conclusion: It follows from this definition that:

1. For each quadratic matrix A, a matrix,

there are, respectively, the main diagonal, the non-main diagonal, the vertical axis, the horizontal axis, and the center of the matrix.

If A is a square matrix, then,

is the decomposition of the given matrix A into the sum of symmetric and conical matrices relative to the main diagonal, the non-main diagonal, the vertical axis, the horizontal axis and the center of the matrix, respectively.

In construction, the matrix is symmetrical, the matrix is skew-symmetrical relative to the main (non-main) diagonal, the vertical (horizontal) axis, and the center of the matrix, respectively.

2. If the square matrix A is skew-symmetric about

1) the main diagonal, 2) the non-main diagonal, 3) the vertical axis, 4) the horizontal axis, 5) the center of the matrix, then it can be broken down into block matrices of the form:

1) at n=2k at n=2k+1

2) at n=2k at n=2k+1

3) at n=2k at n=2k+1

4) at n=2k at n=2k+1

5) at n=2k at n=2k+1

where all block matrices are square k-th order

.

The validity of this statement follows from definition 2 and definition 6.

Definition 7. The square matrix of order n is called orthogonal with respect to:

1) the main diagonal, if ,
2) non-main diagonal, if ,
3) the vertical axis, if ,
4) horizontal axis, if ,
5) the center of the matrix, if .

Thus, the concept of generalized matrix transpositions is introduced and their properties are investigated, the geometric and mechanical meaning of generalized transpositions is discussed. Correspondence between the matrix structure and the LSS structure has been established. According to the generalized transposition of matrices, different types of symmetric matrices are established.

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